Week 3.

Homework: Due Wednesday September 28.

Read p.30-43 of Rudin.

Problems to turn in

Problem 1: Suppose X is a metric space with a metric d_X. Prove that if Y is a non-empty subset of X, then Y can be made into a metric space with the metric d_Y defined by

$$d_Y(y_1, y_2) = d_X(y_1, y_2).$$

Problem 2: Suppose X and Y are two metric spaces with metrics d_X and d_Y, respectively. Show that the Cartesian product $Z = X \times Y$ of X and Y becomes a metric space with the following distance function

$$d_Z((x_1, y_1), (x_2, y_2)) = d_X(x_1, x_2) + d_Y(y_1, y_2).$$

Recall that the Cartesian product of two sets A and B is the set of ordered pairs (a, b) where $a \in A$ and $b \in B$.

Problem 3: Show that every non-empty set X may be turned into a metric space by defining a distance function on X by setting $d(x, y) = 1$ if $x \neq y$ and $d(x, x) = 0$.

Problem 4: Let S be the set of sequences of real numbers $\{a_i\}$ such that the set

$$\{ \sum_{i=1}^{N} a_i^2 \; : \; N = 1, 2, 3, \ldots\}$$

is bounded above. Define a distance on S as follows

$$d(\{a_i\}, \{b_i\}) = \left(\sup\{ \sum_{i=1}^{N} (a_i - b_i)^2 \; : \; N = 1, 2, \ldots\} \right)^{1/2}$$

Prove that the function d is well-defined. Show that S endowed with the metric d becomes a metric space.

Do problems 7, 9, 11 on page 43 of Rudin.