HOMEWORK 6

You may work on the problem set in groups; however, the final write-up must be yours and reflect your own understanding. In all these exercises assume that k is an algebraically closed field and R is a commutative ring with unit.

Problem 0.1. *Calculate the Hilbert polynomial of a linear space of dimension k in $\mathbb{P}^n$.*

Problem 0.2. *Calculate the Hilbert function of the rational normal curve of degree d given by*

$$[x_0, x_1] \to [x_0^d, x_0^{d-1}x_1, \ldots, x_1^d]$$

by noting that the homogeneous polynomials of degree m in the coordinates of $\mathbb{P}^d$ pull-back to give all homogeneous polynomials of degree md in two variables. More generally, using the same observation show that the Hilbert function of the d-th Veronese image of $\mathbb{P}^n$ is given by

$$h(m) = p(m) = \binom{md+n}{n}.$$

Problem 0.3. *Calculate the Hilbert polynomial of a hypersurface of degree d in $\mathbb{P}^n$.*

Problem 0.4. *Calculate the Hilbert polynomial of a pair of skew lines in $\mathbb{P}^3$. Calculate the Hilbert polynomial of a pair of intersecting lines in $\mathbb{P}^3$.*

Problem 0.5. *Calculate the Hilbert polynomial of three concurrent lines in $\mathbb{P}^3$ that do not lie in a plane. Calculate the Hilbert polynomial of three concurrent lines in $\mathbb{P}^3$ that do lie in a plane. Are these closed algebraic sets isomorphic?*