HOMEWORK 7

You may work on the problem set in groups; however, the final write-up must be yours and reflect your own understanding. In this problem set assume that all the varieties are defined over the complex numbers.

Problem 0.1. *Let X be a smooth, irreducible, non-degenerate (i.e., does not lie in any hyperplane) curve in $\mathbb{P}^3$. Show that the union of the tangent lines to X is a projective surface. Calculate this surface S when X is a twisted cubic. What is the singular locus of S in this case?*

Problem 0.2. *Let X be a smooth curve in $\mathbb{P}^2$. A line l is called a flex line to X if it has contact of order three at a point $x \in X$. Find the flex lines to the curve $x^3 + y^3 + z^3 = 0$ (hint: there are nine of them). How many of these flex lines are defined over the real numbers?*

Problem 0.3. *Show that a smooth, irreducible curve has finitely many flex lines.*

Problem 0.4. *Show that the projective tangent space to the Veronese variety $v_d(\mathbb{P}^n)$ in $\mathbb{P}^{\binom{n+d}{d}-1}$ at a point L^d may be described as the set of polynomials of degree d divisible by L^{d-1}. Conclude that the Veronese varieties are smooth.*

Problem 0.5. *Let X be a smooth plane curve. Define a map $X \to \mathbb{P}^{2*}$ by sending $x \in X$ to the tangent line to X at x. The image of X under this map is called the dual curve. Show that the dual curve to a smooth conic $x^2 - yz$ is itself a smooth conic.*