MATH 417 HOMEWORK 6

This homework is due Wednesday October 8 in the beginning of class. You may collaborate on the homework. However, the final write-up must be yours and should reflect your own understanding of the problem. Please be sure to properly cite any help you get.

Problem 1 Calculate the following integrals where C is the positively oriented boundary of the square with vertices at $2-2i, 2+2i, -2+2i, -2-2i$

(1)

$$\int_C \frac{e^{-z}}{(z-i)} dz$$

(2)

$$\int_C \frac{\cos(z)}{z(z^2+25)} dz$$

(3)

$$\int_C \frac{z^2+8}{2z-1} dz$$

Problem 2 Evaluate the following integrals along the contour $|z-i| = 2$ oriented in the positive sense.

$$\int_C \frac{dz}{z^2+4}, \quad \text{and} \quad \int_C \frac{dz}{(z^2+4)^2}$$

Problem 3 Let C be a simple closed contour oriented positively. Let f be analytic in a domain containing C and its interior. Let $f^{(n)}$ denote the n-th derivative of f with respect to z. Let z_0 be in the interior of C. Show that

$$f^{(n)}(z_0) = \frac{n!}{2\pi i} \int_C \frac{f(z)dz}{(z-z_0)^{n+1}}.$$

Problem 4 Let C be the unit circle. Show that for any real constant a

$$\int_C \frac{e^{az}}{z} dz = 2\pi i.$$

Deduce that

$$\int_0^{\pi} e^{a\cos(\theta)} \cos(a\sin(\theta)) d\theta = \pi.$$

Problem 5 Show that if C is a positively oriented simple closed contour, then the area of the region enclosed by C is given by the integral

$$\frac{1}{2i} \int_C \overline{z} dz$$

(Hint: Use Green's theorem.)