MATH 417 HOMEWORK 7

This homework is due Wednesday October 22 in the beginning of class. You may collaborate on the homework. However, the final write-up must be yours and should reflect your own understanding of the problem. Please be sure to properly cite any help you get.

Problem 1 Find the Taylor series expansion of the following functions around $z = 0$. State the radius of convergence of the series.

(1) $z \cos(z^2)$

(2) e^{3z^3}

(3) $\frac{9z}{1-4z^3}$

Problem 2 Find the radius of convergence of the following power series:

$$(a) \sum_{n=1}^{\infty} n z^n \quad (b) \sum_{n=1}^{\infty} \frac{z^n}{n} \quad (c) \sum_{n=1}^{\infty} \frac{z^n}{2^n} \quad (d) \sum_{n=1}^{\infty} \frac{z^n}{n^n} \quad (e) \sum_{n=1}^{\infty} z^{n^2}$$

Problem 3 Find elementary expressions for the following power series

$$(a) \sum_{n=0}^{\infty} z^{3n+1} \quad (b) \sum_{n=2}^{\infty} n(n-1)z^n \quad (c) \sum_{n=1}^{\infty} \frac{z^{2n}}{n!}$$

Problem 4 Find the Laurent series expansion of

$$f(z) = \frac{1}{z^3} \cos(\frac{1}{z^2})$$

valid in the region $0 < |z| < \infty$.

Problem 5 Give two Laurent series expansions in powers of z for the function

$$f(z) = \frac{1}{z(1-z^2)}$$

and indicate the regions where they are valid.