Math 215: Problem set 1

Due 1/26

1. Prove that for three distinct real numbers $a, b, c \in \mathbb{R}$,

$$\frac{a^4}{(a-b)(a-c)} + \frac{b^4}{(b-a)(b-c)} + \frac{c^4}{(c-a)(c-b)} = a^2 + b^2 + c^2 + ab + bc + ac.$$

2. * Show that if a, b and c are positive real numbers, then

$$(a^2 + b^2)c + (b^2 + c^2)a + (a^2 + c^2)b \geq 6abc.$$

3. Let a, b and c be positive real numbers.

   (a) Show that
   $$\frac{ab}{a+b} \leq \frac{a+b}{4}.$$

   (b) * Show that
   $$\frac{ab}{a+b} + \frac{bc}{b+c} + \frac{ac}{a+c} \leq \frac{a+b+c}{2}.$$

4. Let α and β be positive real numbers. Show that $\alpha \leq \beta$ if and only if $\sqrt{\alpha} \leq \sqrt{\beta}$.

5. *[1] Show that $\max\{a, b\} = \frac{a+b+|a-b|}{2}$.

6. Prove that $x^2 - 3x + 5 \geq 0$ for all real numbers x.

[1] Each week, there will be one or more problems marked with a *. These are the problems you must write up carefully and turn in during the class period on the due date. The other problems should also be done - they are the prime candidates for quiz problems.