Math 215: Problem set 2

Due 1/26

1. * Let $a_1, \ldots, a_n$ be positive real numbers. Prove that

$$\min_{1 \le i \le n} a_i \le \frac{n}{\sum_{i=1}^{n} \frac{1}{a_i}} \le \max_{1 \le i \le n} a_i.$$

2. Let a_1, a_2 be positive real numbers. Prove that

$$\frac{1}{\frac{1}{a_1} + \frac{1}{a_2}} \le \sqrt{a_1 a_2} \le \frac{a_1 + a_2}{2}.$$

3. * Prove that for all $a, b \in \mathbb{Z}$,
$$a^2 - 4b \ne 2.$$

4. Prove that for all $x \in [0, \pi/2]$,

$$\sin(x) + \cos(x) \ge 1.$$

   (Feel free to use standard facts from trigonometry.)

5. * Prove that $\sqrt[3]{2}$ is irrational.

6. Show that there are no integers $a, b \in \mathbb{Z}$ such that $24a + 9b = 1$.