# HW9, Math 506, Fall 2016, Due 12/10

November 30, 2016

Throughout the assignment, assume $T$ is a complete theory in a countable language which has infinite models. Let $\mathcal{M}$ be a saturated enough model of $T$.

1. (2 points) Let $\mathcal{N} \models T$. Show that $\mathcal{N}$ is an extension base.
2. (2 points) Let $T$ be an arbitrary theory with Skolem functions. Let $A \subset \mathcal{N} \models T$. Show that $A$ is an extension base.
3. (2 points) Find a theory $T$ and a type over $\emptyset$ such that $p$ is definable, but there is no definition of $p$ which defines a global type. (It would be wise to think about the requirements on the theory given what we proved in class.)
4. (2 points) The Morley rank of $T$ is defined to be the Morley rank of the formula $x = x$ where $x$ is a singleton. Find a theories of Morley rank $n$ for each $n \in \mathbb{N}$.
5. (2 points) Consider the definable set in a model of $ACF_0$ given by an algebraic surface. Show that the Morley rank of the the surface is two.
6. (4 points) The Cantor-Bendixson derivative of a compact Hausdorff space, $X$, is denoted $\Gamma(X)$, and is defined:

$$\Gamma(X) = \{x \in X \,|\, x \text{ is not an isolated point}\}.$$

   We define the iterated derivative on $X$ as follows. Define $\Gamma^\alpha(X)$ via $\Gamma^0(X) = X$, $\Gamma^{\alpha+1}(X) = \Gamma(\Gamma^\alpha(X))$, and if $\alpha$ is a limit, then $\Gamma^\alpha = \cap_{\beta<\alpha}\Gamma^\beta(X)$.

   - Show that $\Gamma^\alpha(X)$ is closed.
   - Show that there is an ordinal $\delta$ such that $\Gamma^\delta(X) = \Gamma^\alpha(X)$ for any $\alpha > \delta$.
   - If $X$ is separable, show that $\delta < \omega_1$.
   - If $X$ is separable, show that if $\Gamma^\delta(X)$ is not empty, then $|\Gamma^\delta(X)| = 2^{\aleph_0}$.
   - Let $X$ be $S_n(\mathcal{M})$ and suppose that $T$ is $\omega$-stable. Define the Cantor-Bendixson rank of a type $p$ to be the ordinal $\alpha$ such that $p \in \Gamma^\alpha(S_n(\mathcal{M})) \setminus \Gamma^{\alpha+1}(S_n(\mathcal{M}))$. Show that the Cantor-Bendixson rank of a type is equal to the Morley rank of the type.