HW8, Math 506, Fall 2016, Due 11/18

November 14, 2016

Throughout the assignment, assume T is a complete theory in a countable language which has infinite models. Let $\mathcal{M}$ be a saturated enough model of T.

1. (5 points) Consider the structure whose elements are points on a circle centered at the origin in the plane $\mathbb{R}^2$. Let the relation $R(x, y, z)$ hold in the structure if y is between x and z considered clockwise.
    1. Prove that the theory of this structure eliminates quantifiers.
    2. Show that there is a unique 2-type over the empty set which includes the formula $x \neq y$.
    3. Show that $R(a, y, c)$ divides over the emptyset for any a, c.
    4. Show the formula $x \neq x'$ forks over the empty set.
    5. Explain why no formula can divide over its own parameter set.
2. (1 point) Let $p(x) \in S_x(\mathcal{M})$ be some complete type. Assume that $p(x)$ does not divide over some small set A. Show that $p(x)$ does not fork over A.
3. (3 points) Let $A \subset B$. If a type $q \in S(B)$ is definable over A or finitely satisfiable in A, then show that q *does not split* over A (we say that q *does not split* over A if for all $a \equiv_A a'$ from B and $\varphi(x, y) \in \mathcal{L}(A)$ we have that $\varphi(x, a) \in q$ if and only if $\varphi(x, a') \in q$).
4. (3 points) Let $A \subset B$. Suppose that $A \models T$, and $q \in S(B)$ is definable over A, then q is an heir over A.
5. (3 points) Let $A \subset B$. Suppose that $B = \mathcal{M}$ and $q \in S(B)$ is A-invariant. Then q does not fork over A.
6. (3 points) Let $\mathcal{N}$ be a small model of T. Show that $tp(a/\mathcal{N}b)$ is an heir of $tp(a/\mathcal{N})$ implies that $tp(b/\mathcal{N}a)$ is a coheir of $tp(b/\mathcal{N})$.