Mathematic 300, Spring 2003

Short Writing Exercise 2
Pictorial proof

Below is a pictorial proof of the following formula for the sum of a geometric series.

$$1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^n} + \cdots . = 1 \tag{1}$$

Picture can be found at: http://www.math.uic.edu/ radford/Short2.pdf

1. Write a proof of this equation based on this pictorial argument, but using no pictures. You may use mathematical symbols and equations. Write the proof as if you were explaining it to a friend who must understand it only from your verbal explanation.[1]

2. Now consider the equation:

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} + \cdots . \tag{2}$$

   Explain how to add to the picture given above to show that this series has a finite limit. Do not attempt to find the exact value.

[1] Adapted from picture by David Radford and page 118 of Proofs without Words by R.B. Nelson, M.A.A., 1993, where the idea is attributed to Warren Page.

3. You may choose to incorporate this assignment into essay 1. If not, a separate typed solution to this exercise is due Feb. 17. The outline for Essay 2 is also due on February 17.