3.2 Do the Cuts Really Work?

Have you ever expected a dissection to work, but then discovered that the pieces did not quite fit? Or perhaps the pieces looked like they fit, but you found it difficult to be sure?

To understand why a dissection works, you must know properties of the shapes you are cutting. Here are some properties of parallelograms that you learned in Chapter 2.

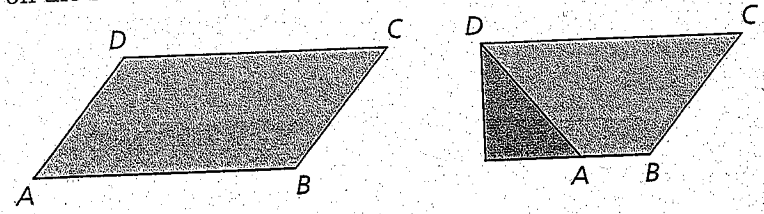
- Parallelograms have exactly four sides.
- Opposite sides are parallel.
- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Habits of Mind

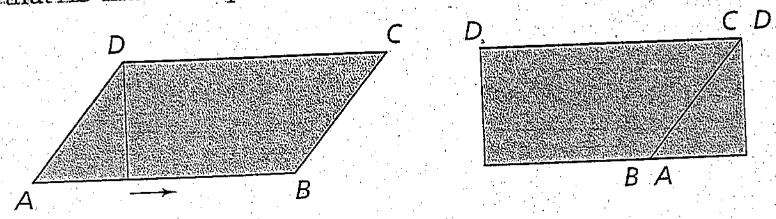
Confirm a process. If you can explain *why* a cut works, then you will know for sure that it does.

Here is Tony's method for dissecting a parallelogram into a rectangle.

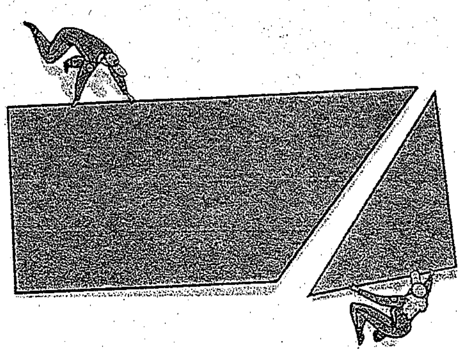
Cut out the parallelogram. Make a fold through vertex D so that A lines up on the bottom, on $\overline{AB}$.

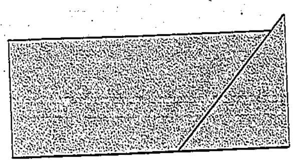


Then unfold and cut along the crease. Slide the triangular piece along $\overline{AB}$ so that $\overline{AD}$ matches up with $\overline{BC}$ and you have a rectangle.

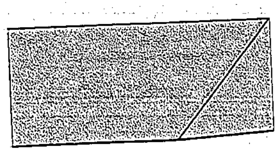


Tony cut two pieces: a triangle and a trapezoid. He then rearranged those pieces. But what guaranteed that the rearrangement had four sides? Here are two ways that his dissection might fail.





The newly glued edges might not match.



The new bottom edge might be crooked.

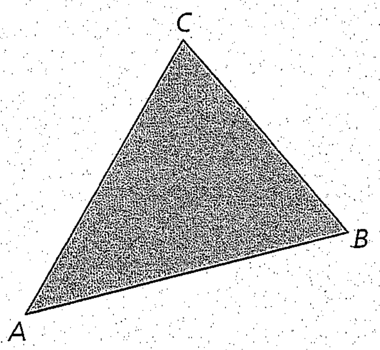
For Discussion

In Tony's method on the previous page, you slide the triangle to the opposite side of the trapezoid. Explain how the properties of parallelograms guarantee each of the following.

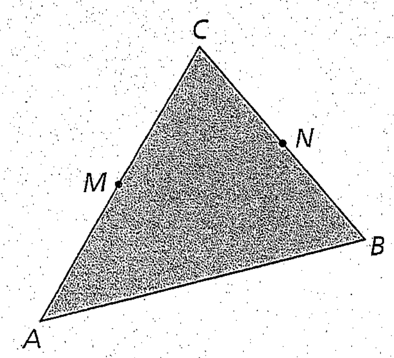
1. The two pieces fit together exactly.
2. The new bottom edge is straight.

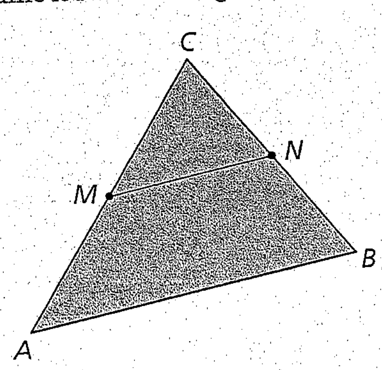
Minds in Action episode 5

Sasha and Derman are trying to dissect $\triangle ABC$ into a parallelogram.

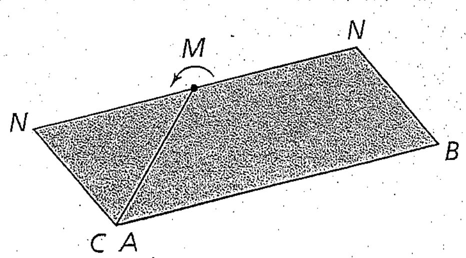


Sasha First, cut out the triangle. Find the midpoint of $\overline{AC}$. Name it M. Find the midpoint of $\overline{BC}$. Name it N. Cut along $\overline{MN}$.





Derman I see where you're going with this. You can rotate $\triangle MCN$—the triangle you just cut off—around M, until $\overline{MC}$ matches up with $\overline{MA}$.



Sasha Now we have a parallelogram, right?