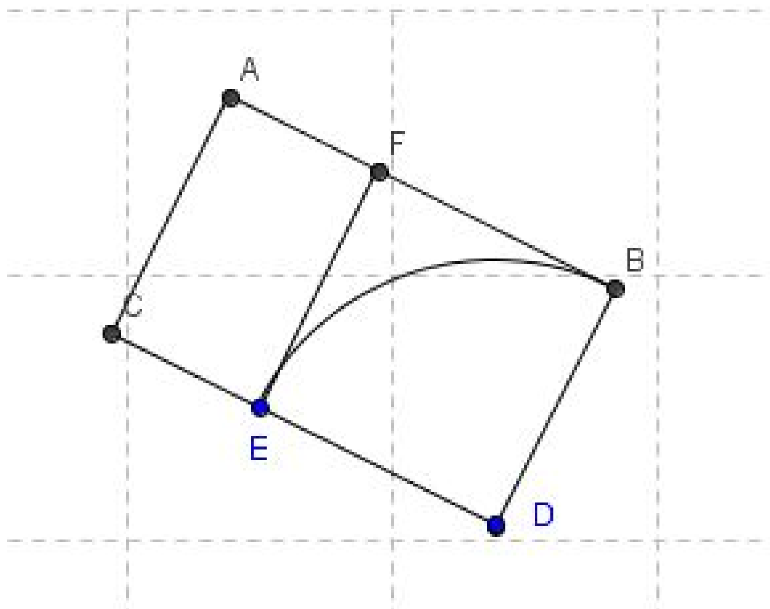
The Golden Ratio*

CTTI

December 1, 2012

1. In the rectangle below: $BD = ED$, $FE \parallel BD$ and $\frac{BD}{CD} = \frac{EC}{AC}$.

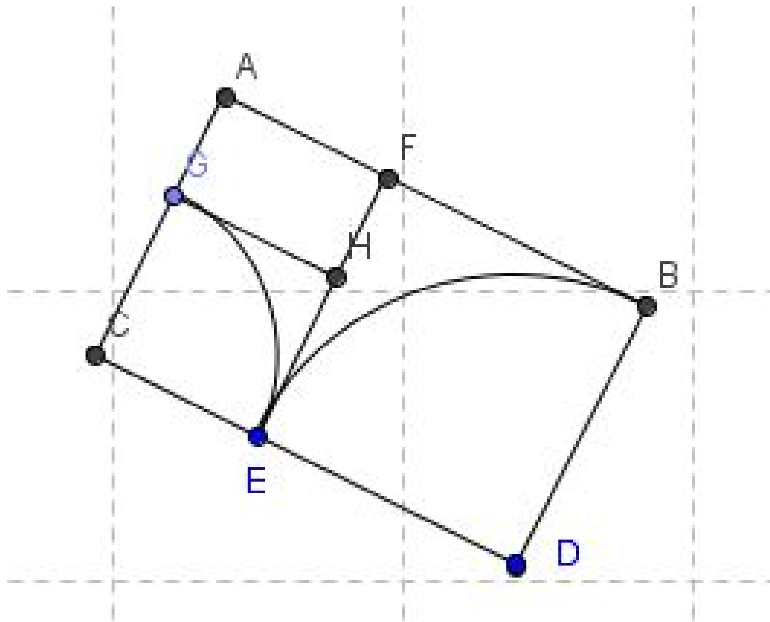
   If BD has length 1 and CD has length ϕ, compute ϕ.



2. I used the letter ϕ to denote the ratio that holds when $\frac{a}{b-a} = \frac{b}{a}$ where a and b are the altitude and base of a rectangle. Do you know why? What are the advantages and disadvantages of this notation?

*CTTI Dec. 3, 2012, adapted from Smorynski- History of Math, A supplement

3. Continue the diagram by choosing G, H so that $CE = CG$ and GH is parallel to CD.



So $\frac{BD}{CD} = \frac{EC}{AC} = \frac{AG}{AF}$.

(Verify this statement by calculating the lengths using what you know about ϕ; it is easier if you use the equation $\phi^2 - \phi - 1 = 0$ instead of the actual value of ϕ.)

4. What does the word commensurable mean? Can you think of synonym to explain for high school students. Is it part of the current geometry curriculum? It doesn't appear in CCSSM; Should it?

5. Could it be the case that there are integers m, n such that $\phi = \frac{m}{n}$. That is, is it possible that there are points X on BD and X' on CD such that $BX \cong CX'$ and n copies of BX make BD and m copies of CX make up CD.

   (Hint: Think of what this means for the successive rectangles, $ACBD, CEFA, GHFA$. More precisely, suppose $m_0, m_1, \ldots$ are the number of disjoint segments congruent to BX in the long side of the i rectangle. What can you say about the m_i?)

   Note that the relation $\frac{a}{b-a} = \frac{b}{a}$ implies that in CE is $m - n$ copies of CX and AC is n copies of BX. That is at each stage the unit remains the same. So in fact the construction is guaranteeing that all sides of the successive rectangles are commensurable with the *same* unit.