

Geometry and
Proof

John T.
Baldwin

Hilbert's
Critique

Three
Frameworks

High School
Curriculum

Geometry and Proof

John T. Baldwin

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<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-02-825275-6chapter=2>

Outline

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1 Hilbert's Critique

2 Three Frameworks

3 High School Curriculum

CONTEXT

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How does the axiomatization of geometry affect the teaching of high school geometry?

CONTEXT

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How does the axiomatization of geometry affect the teaching of high school geometry?

The U.S. as a case study. What happens elsewhere?

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- 1 Undefined Terms
- 2 Continuity Axioms
- 3 The Mobility Postulate

Undefined terms

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Two kinds of definitions:

- 1 The 'system' of basic notions, not the individual notions, (points, lines, etc) is defined.
- 2 But auxiliary notions are introduced as abbreviations.

Undefined terms

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Two kinds of definitions:

- 1 The 'system' of basic notions, not the individual notions, (points, lines, etc) is defined.
- 2 But auxiliary notions are introduced as abbreviations.

This distinction is routine for logicians but unknown to high school teachers.

Continuity Axioms

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The continuity axioms leads to 'geometry over the reals'.
'Coordinatizing Ring' is a foreign notion to the Greeks.

Continuity Axioms

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The continuity axioms leads to 'geometry over the reals'.
'Coordinatizing Ring' is a foreign notion to the Greeks.

How do you explain similarity of figures whose side lengths are
incommensurable?

Superposition

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Heath points out a long history of criticisms of Euclid use of superposition to prove the congruence theorems.

Hilberts solution: Assume only SAS

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- 1 Euclid
- 2 Hilbert
- 3 Birkhoff/Moise

Euclid

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Undefined Terms

points, lines, planes

Basic Relations

incidence, congruence,

Defined Relations

addition, multiplication

Axioms

(omitted continuity, 'sneaked in' superposition, no explicit congruence axioms)

Hilbert

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Undefined Terms

points, lines, planes

Basic Relations

betweenness, congruence

Defined Relations

addition, multiplication

Axioms

adds continuity, SAS

Birkhoff/Moise

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Undefined Terms

points, lines, planes, real numbers,

Basic Relations

length functions, angle measure functions, plus, times

Defined Relations

congruence (of segments, angles, figures)

Axioms

real number axioms; correspondence of geometry and numbers,
SAS

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The Birkhoff-Moise framework is almost universal.
One goal is to integrate algebra and geometry.
Another was to avoid the 'errors' of Euclid.

Difficulties with current curriculum

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- 1 Euclid's early propositions have real proofs; the basic facts of algebra are trivialities.

Difficulties with current curriculum

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- 1 Euclid's early propositions have real proofs; the basic facts of algebra are trivialities.

Difficulties with current curriculum

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- 1 Euclid's early propositions have real proofs; the basic facts of algebra are trivialities.
- 2 Problem: Students can't do (algebra) proofs.

Difficulties with current curriculum

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- 1 Euclid's early propositions have real proofs; the basic facts of algebra are trivialities.
- 2 Problem: Students can't do (algebra) proofs.
- 3 Solution: Take (geometry) proofs out of the curriculum.

STUPID PROOF

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Flattening out Geometry

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An 'honors' text in the U.S. has 24 postulates including:
SAS, SSS, ASA, HL,
3 (ruler, protractor, segment addition) tie geometry to unstated
axioms for real arithmetic

The role of Proof

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Proof is still a goal of state standards. But the textbooks are not adequate for students to learn how to prove.
There are many reasons; I focus on the mathematical one.

Diagnosis

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The fundamental problem is:
How do we come to grips with congruence and similarity?

Can one resurrect the principle of superposition?

Another Approach (Weinzweig/Hartshorne)

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Undefined Terms

points, lines, planes, rigid motions

Basic Relations

incidence, application of rigid motions

Defined Relations

congruence, addition, multiplication

Axioms

properties of rigid motions and basic geometry

What I learned in High school geometry

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What I learned in High school geometry

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DON'T TRUST THE PICTURE!

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DON'T TRUST THE PICTURE!

This is often undermined by the use of tools like Geometric Sketchpad. But,

Geometric Sketchpad

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Draw a quadrilateral; connect the midpoints. What do you get?

Lessons for Preparing Teachers

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The goals of proof are

- 1 not the mere verification of truth
- 2 but the gaining of understanding

Proof is a more efficient way retaining information than memorization.