Math 215: Assignment due Apr. 17

1. Suppose $x + 1/x = 7$; compute $x^2 + 1/x^2$ and $x^3 + 1/x^3$. Prove that if $x + 1/x$ is an integer so is $x^n + 1/x^n$.

2. Show $\Sigma_{k=0}^{k=n} \binom{n}{k} = 2^n$. (Hint: Use the binomial theorem.)

3. Consider the real numbers less than 10 represented as infinite decimals $a_0.a_1a_2a_3....$ with $a_0.a_1a_2a_3.... < b_0.b_1b_2b_3....$ defined as usual.

   (a) Show between any two such (distinct) real numbers there is a third.

   (b) Show between any two such (distinct) real numbers there is a rational number. (Rational is to be taken as repeating decimal.)