

**Math 215: Introduction to Advanced Mathematics**  
Last Problem Set

**Due Tuesday May 1**

Recall that  $n$  is divisible by  $d$  if there is a  $q$  with  $qd = n$ .

1. page 225 number 2, page 271: 1,3
2. Assume the division algorithm for the natural numbers. If  $a, b$  are integers with  $a \geq 0$  and  $b > 0$  there are unique (positive) integers  $q, r$  with  $0 \leq r < b$  such that:

$$a = qb + r.$$

Complete the proof for negative  $a$ . What are  $q$  and  $r$  if  $a = -123$  and  $b = 7$ ?

3. Recall the two definitions I gave on April 24.  
( $N, <, +$ ) satisfies IND if for every  $A \subset X$ : If  $1 \in A$  and  $k \in A$  implies  $k + 1 \in A$  then  $A = N$ .

( $N, <, +$ ) satisfies WO if every nonempty  $A \subset X$  has a least element.

I proved in class that WO implies IND. Show IND implies WO. (Note of course that both of these are actually true of the natural numbers.)