A Proof by Induction

Theorem: In a game with scores of 4 and 3, 5 is the greatest impossible number. That is, if $n > 5$, there exist a and b such that $4a + 3b = m$.

Note that 5 cannot be represented in the form $4a + 3b/$

Let $P(k)$ be the proposition: for every m with $5 < m \leq k$ there exist a and b such that $4a + 3b = m$.

Proof. We will prove by induction on $n \geq 8$ that $P(n)$ holds. We need $P(8)$.

Claim 1: $P(13)$ holds.

Check Claim 1: $6 = 2 \cdot 3$, $7 = 4 + 3$, $8 = 2 \cdot 4$.

Claim 2: P(k) implies P(k+1).

Proof of Claim 2. Let $m = (k+1)-3$. By induction there exist there exist a' and b' such that $4a'+3b' = m$. That is, $4a'+3b' = k-2$. So, $4a'+3b'+3 = k+1$. So if we set $a = a'$ and $b = b' + 1$, $4a + 3b = k + 1$.

Since for any k, we have shown $P(k)$ implies $P(k + 1)$ by the mathematical induction we have shown: for all n, $P(n)$.