Study Guide to Second Midterm

March 11, 2007

Name:

Several of these were review problems for the first midterm. If you did well on the midterm, you may not have thought about them. There are more problems here than will be on the exam. The exam is cumulative; you are responsible for concepts from the first exam. I will post solutions to those problems in which the class show sufficient interest.

1. Make sure you know and can use the definitions of such notions as symmetric difference, union, power set, injection, surjection, bijection, finite ....

2. Show explicity using the axioms of order that if $a < b$, $c < d$ are real numbers then $(a, b) \cup (c, d) = (c, d)$ iff $c \leq a$ and $d \geq b$.

3 Prove from the axioms on ordered fields that there is no greatest real number a such that $a^2 < 2$. (Hint: Use that $\sqrt{2}$ is a real number. This does *not* need any complicated algebra.)

4 You may assume basic algebraic properties without explicit reference to the axioms in this problem. Recall that for integers d, s, n, 'd divides n means there is an s such that $n = ds$. Let d, x and y be integers. Prove

   (a) If d divides x and d divides y then d divides $x + y$.

   (b) If d divides $x + y$ and d divides y then d divides x.

   (c) Explain the connection between the two results.

3. True or false: if f is a surjection from X to Y then f has an inverse.

4. Draw Venn diagrams and use an element proof to show: $A \cup (A \cup B) = A$.

5. Let $f : X \to Y$ and $g : Y \to Z$. Suppose $g \circ f$ is surjective.

   (a) Show g is surjective.

(b) Show by example that f does not need to be surjective.

6. Does the function $e^x : \Re \to \Re$ have an inverse $\ln : \Re \to \Re$? If not choose an appropriate domain and codomain so a restriction of e^x does have an inverse. Why did you make the specific choice of domain and codomain.