Sample First Midterm Math 215

1. Consider the statement: If $a < b$ then $f(a) < f(b)$.

    1. What is the contrapositive of the statement?

    2. What is the converse of the statement?

    3. What is the negations of the statement?

2.

    1. Write the truth table for $(P \to Q) \to (Q \to P)$.

    2. Find a simpler statement equivalent to $(P \to Q) \to (Q \to P)$.

3. Prove from the axioms of ordered fields or give a counterexample to each of the following.

    1. $x < y$ implies $x^3 < y^3$.

    2. $x < y$ implies $x^4 < y^4$.

4. Prove that if $a^2 \geq 7a$ then $a \geq 7$ or $a \leq 0$. Does the converse also hold?

5. Prove that $\prod_{i=2}^{n}(1 - \frac{1}{i^2}) = \frac{n+1}{2n}$ for all $n \geq 2$. Why did we start with $i = 2$?