SOME PARTIAL SOLUTIONS

1. Similar to Problem 10, page 116 but maybe slightly harder: If $(a, \infty) \subseteq (b, \infty)$ then $a \geq b$.

To save typing I write A for (a, ∞) and B for (b, ∞). Note that A and B are *sets* of real numbers while a and b are real numbers.

I must show $A \subseteq B$ implies $a \geq b$. Now $A \subseteq B$ means for all x, if $x \in A$ then $x \in B$. By definition $x \in A$ means $x > a$ and $x \in B$ means $x > b$. So $A \subseteq B$ translates to: for all x, if $x > a$ then $x > b$. If, for contradiction it is not the case that $a \geq b$, we would have $b > a$. Applying our translation, we conclude, from the transitivity of the inequality relation that $b > b$. This is impossible. So we conclude $a \geq b$.

Remark. 1. The key point that involves the real numbers is the transitivity property of the ordering. There needs to be some reference to the properties of inequality to give a complete proof, although you might be less formal about it.

2.To say x is a value of A instead of x is an element of A or $x \in A$ expresses the right idea but is not the conventional notation and will often be misunderstood.

2. The following problem was number 5 on the homework for Feb. 27.

Prove by truth tables the following two equivalences.

A.

$$(p \rightarrow q) \leftrightarrow (\neg p \vee q).$$

(I.e. in English, (p implies q) if and only if ((not p) or q.

B.

$$(\neg(p \rightarrow q) \leftrightarrow (p \wedge \neg q)$$

(I.e. in English, (not(p implies q)) if and only if (p and not q).

C. Use this information to write the negation of the proposition.

if $f(a) < g(b)$ then $a < b$.

(It originally said $f(a) < g(a)$ – that introduces unnecessary confusion although it does not really change the problem. But I have rewritten it so it is easier to understand.

Most of you got A and B correct but C wrong. Note that C asks you to write in a shorter form the meaning of: It is not the case that if $f(a) < g(b)$ then $a < b$.

HINT: Use part B.

3. Give an example of a set A such that $A \not\subseteq P(A)$.