Essay I

Compound Interest and Mortgages

John Baldwin[1], January 2009

Due dates:

Jan 28: First draft of Essay 1 (week 3).

February 18: Final draft of Essay 1 (week 6).

The Harvard-on-Halsted school of Journalism offers a course called: Writing for the Financial Pages. This course assumes that students are familiar with basic financial mathematics of interest and mortgages as taught in a pre-calculus or business mathematics course.

One of the assignments is to write a newspaper article on understanding compound interest and mortgages. Printed below is a draft of a student's essay. The draft is too short, poorly written, unclear, and filled with errors. You are to completely rewrite this draft so that it will understandable to a person with a good understanding of high school algebra. Your essay should cover the same material as in the draft but possibly in a different order. In particular, your essay should have an introduction, explain to the reader what compound interest is, distinguish simple, compound, and continuous interest, present the formula for compound interest, carefully justify why the formula works, give a dramatic example of how compound interest can increase the value of an investment, discuss the rule of 72, give some examples of the use of the rule of 72, and prove why the rule of 72 works. The article should include examples of the role of compound interest in making investments and in buying a house.

The audience for your essay is a newspaper reader. You may assume the readers of the article know the rules for exponents and how to convert percent to decimal. But you will have to decide how to convey information about the natural logarithm ln and the number e and the relationship between e and ln to the general public.

Here is one way to type subscripts and superscripts in Word. First type the text normally. Then highlight the symbol or string of symbols that are to be superscript or subscript. Click on Format and then click on Font. Check the box for superscript or subscript and the highlighted item will be changed. The keyboard commands for this are `CTRL =` and `CTRL +`

The symbol $\approx$ denotes "approximately equal". To type this in Word, select Symbol from the Insert menu. A rectangular grid of symbols will appear on the screen. Scroll through these symbols to find the rows containing the mathematical symbols. The symbol $\approx$ should be among them. Highlight $\approx$ in the grid and click on Insert and then click on Close. The symbol will then appear in the text at the cursor. This method can be used to print all the other symbols available on this menu.

Draft of Essay I: Compound Interest and Mortgages

Compound interest is without a doubt the most important thing in mathematical finance. In this article we see why.

The formula for compound interest is $A = P(1 + r)^n$ where P is the initial amount of money or principal, A is after n time periods, and r will be the interest rate per time period. For example, if the compounding period is one year and the money is deposited for four years and $P = \$516$ and $r = .06$, then $n = 4$ and the formula gives $A = \$651.44$. If instead

[1] Adapted from materials prepared by Joel Berman

you go ahead and then make the compounding period to be one month in this example, then $r = .005, n = 48$ and $A = \$655.57$.

The formula works because, after the first time period the amount A becomes $A = P + Pr = P(1+r)$. At the end of the second time period the amount is $A = P(1+r) + P(1+r)r = P(1+r)^2$. And so on. So $A = P(1+r)^n$ for all positive integers n is the general formula.

Example: \$5000 is deposited in a savings account at a large bank in a small town near Chicago. The population of the town is 6500. The account pays 4% interest per year compounded monthly. The money is left undisturbed in this account for ten years. During this time the value of the U.S. dollar rises 8% against the Japanese yen. After ten years by how much, in dollars, has the account increased in value?
Solution: Clearly P is 5000. It is absolutely trivial to see that $r = .04/12 = .0033$. So to find n you just multiply 12 by 10 and I get 120 which we know to be n and she can use this value of n in the formula. So $A = 5000(1+.00333)^{120} = 7451.19$. So the amount is \$7451.19. So the increase after 10 years is \$2451.19.

Example: If in 1790 Benjamin Franklin had invested \$100 at 10% percent interest compounded annually and the amount accumulated undisturbed, then it would be worth a enormous amount of money today.

The Rule of 72: A useful for method for mental computations involving compound interest is called the Rule of 72. It states that the amount of time it takes for an investment to double in value at a compound interest rate of R percent compounded annually is approximately 72/R years. For example, it will take approximately $72/5 = 14.4$ years for money invested at 5 percent to double. The rule can also be used to determine approximately what interest rate is needed to have an investment double in a given number of years. For example, if one wants to know what interest rate is needed to double the principal in 10 years, divide 72 by 10 to obtain the rate of 7.2 percent. Rule: $R = 72/T$.

The justification for the Rule of 72 is that if in the formula $A = P(1+r)^n$ we let A be 2 and P be 1, then n is the number of years for doubling at rate r per year. $2 = (1+r)^n$. As everyone knows, as the positive real number h gets close to 0, the value of $(1+h)^{1/h}$ gets arbitrarily close to e from below. From

$$2 = \left((1+r)^{1/r}\right)^{nr}$$

we get that 2 is approximately equal to e^{nr}, with e^{nr} being an overestimate. Do ln both sides. Use $1 = \ln e$. Get $.6931 \approx nr,\;\; n \approx .6931/r,\;\; n \approx 69.31/R$ for R interest rate r in percent. But this is an underestimate for n. Replace 69.31 by 72 to get a better estimate and to use an integer with many integer divisors to facilitate mental arithmetic by the human brain working quickly on one's feet. That's all, folks.

Example: Estimate the amount of money that \$1000 would grow to at 8 percent per year, compounded annually, for 90 years. By the rule of 72 we see that the money doubles every 9 years. So in 90 years \$1000 grows to approximately \$1,024,000.

Example: The formulas for calculating monthly payments on a mortgage are more complicated because there are more payments. Fortunately, you can find out about this on the internet. For example `http://www.mortgagecalculator.org/` is a good mortgage calculator. To use this tool, you must put in the amount of the loan, the rate per month, and the length of the loan. It gives lots of good information including how much you pay each week. It is very important to know your PMI –ptrerodactl moon index.