

# Notes on final versions of Third Essay

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## 1 Audience

The memo format was required as a way to help you focus on the audience for whom you were writing. Many people missed this aspect of the assignment and wrote with the same tone as the earlier papers that were aimed at a less educated office. These memos were supposedly directed at people who know the product, the Deluxe Set of Blocks. While, some review was fine, a lengthy discussion was not needed.

In particular, the estimate of 10.2 square feet needed to be treated as a step in the right direction. It did not deserve more than a sentence or two.

## 2 Mathematics

Several mathematical misconceptions and miscalculations remained.

Note:

$$\int_m^\infty \frac{1}{x^3} dx = \frac{1}{2m^2}.$$

Several people mistakenly thought the answer was  $\frac{2}{m^2}$ . I deducted very little for this mechanical error.

A more serious problem was to make one of the following two conceptual errors.

1.  $\sum_1^\infty \frac{1}{m^2} \neq 1 + \int_1^\infty \frac{1}{x^2} dx$ .

The right hand side is an upper bound for the left; in fact, it is definitely bigger.

2. Having shown by checking, say  $m = 7$ , that

$$\sum_0^m \frac{1}{n^2} + \int_m^\infty \frac{1}{x^2} dx = 1.66,$$

several students calculated the estimate for larger  $m$  and concluded that because the estimate was still approximately 1.63 that this was 'the value' of the integral.

In general is possible that a function can remain close to a value for a long time and then decrease again. The methods we used only allowed us to provide an upper bound. Just because the calculator gives the same value for a number of approximations, it does not follow that that value is the limit. Thus, phrases like 'seems to converge' were inappropriate in the essay.

Notice that the lower bounds increase very slowly. It is easy to get to 1.51. But the 100th term adds only  $1/10,000$  and in another 900 terms, each term adds less than  $1/1000,000$ .

In addition, it remains important to distinguish between area and surface area. In connecting the surface area of the cubes with integral, the area of one side of the  $n$ th cube is the same as the area of a rectangle with height  $\frac{1}{n^2}$  and width 1.