Assignment 10: due Nov. 12

John T. Baldwin

November 6, 2007

Redoing Descartes

Feel free to e-mail with questions or call 312-343-1897 during week or 312-226-1897 on weekends.

0) There are two Usyskin articles on my website (one by himself; one paired with Wu). We will discuss both next week.

Refer to Oct. 1 notes: http://www2.math.uic.edu/ jbaldwin/math592/geomaxioms.pdf

and to Hilbert's axioms: http://www.math.umbc.edu/ campbell/Math306Spr02/Axioms/Hilbert.html

to do the following problems.

We now go in the opposite direction from the last two weeks. You are given the real field and may assume anything you know about this structure. The goal is to find a model of Hilbert's axioms.

Definition 1 *1. The set of points will be $R \times R$.*

2. A line is formally a pair (b, m), where b is a real number and m is either a real number or the symbol ∞.

3. The point (x, y) is incident to

(a) (m, b) if $y = mx + b$ where m and b are both real or

(b) (b, ∞) if $x = b$

1. Show that in the model we have just defined the axioms I.1, I.2, I.3 of Hilbert are true.

2. Define a relation $B((a_1, a_2), (b_1, b_2), (c_1, c_2))$ to interpret $\mathbf{c}$ is between $\boldsymbol{a}$ and $\mathbf{b}$. Check that your definition verifies axiom group II of Hilbert in the model.

(This may get boring; be sure to do II.1 and II.5 (Pasch). Do as many of the others as you need to.)

3. Prove the parallel postulate (Hilbert III) holds in this model.

Definition 2 *Two segments are congruent if the Cartesian distance between their endpoints is the same.*

4. Prove this model satisfies Hilbert's first three congruence axioms: IV.1, IV.2, IV.3.