

Math 592: Fall 07

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1 Study Guide

Here are a collection of sample questions or topics. Please also read the summary that lays out the themes of the course in more detail.

1. Basic Logic: The following key notions should be known: formal language, vocabulary, sentence, the compositional theory truth, model/structure, truth in a model, validity, consistency, completeness theorem, compactness theorem.

Know the difference between \vdash and \models . Note we first define $M \models \phi$ and then derive the usage $\Gamma \models \phi$.

The major skill is to be able to decide if a particular sentence in a formal language is true in a particular model.

2. proof
 - (a) Give a rigorous definition of the notion of proof in a formal system
 - (b) Translate this into a meaningful explanation for high school students.
3. What are the essential elements of definition?
4. Let the formal language of geometry include: point, line, incident $I(p, \ell)$, between $B(x, y, z)$, congruence \cong (of angles or segments), -where angle ABC and segment AB are defined terms. Give an informal definition using these terms for the following (You may use earlier terms in the list).
 - (a) right angle (answer: Let AB intersect CD at E. Angle AED is a right angle if and only if angle AED is congruent to angle AEC).
 - (b) two angles are supplementary
 - (c) The set of points X is a circle with center A through the point B.
5. Write definitions for each of the terms in 2) in the formal language of geometry. (e.g. AED is a right angle if and only if

$$(\exists C)(\exists \ell)I(A, \ell) \wedge I(C, \ell) \wedge \angle AED \cong \angle DEC.$$

6. What is the difference between a function and a relation; give examples.
7. Give proofs from Hilbert's axioms of such theorems as. Of course you can use any result you have already proved.

- (a) SAS implies ASA: Suppose $AB \cong DE, \angle BAC \cong \angle EDF, \angle ABC \cong \angle DEF$. Choose F' on AC (extended) with $AF' \cong DF$. By SAS $ABF' \cong DEF$. Thus $\angle F'BA \cong \angle FED$ and by hypothesis $\angle BAC \cong \angle EDF$. Thus F' is on BC extended and since two lines meet in a unique point $F' = C$ and we finish.
 - (b) SAS implies SSS (This requires a small construction; check Euclid if you have trouble).
 - (c) If CD intersects AB at E and for each F on $CD, FA \cong FB$ then CD is a perpendicular bisector of AB .
 - (d) The next two are a couple of line proofs just using the definition and the last result.
 - i. The perpendicular bisectors of the sides of a triangle intersect in a single point.
 - ii. Given three distinct points, there is a circle through them.
8. What does it mean for a statement to be independent from a given set of axioms?
9. What pictures should you draw to illustrate the notion of height of a triangle?