

Midterm Exam: MTHT 400
Methods of Teaching Secondary Mathematics
October 3, 2001

John T. Baldwin

October 12, 2005

NAME:

The first 5 problems are mandatory; they are worth 20 points each. Do the problems on your own paper. Be sure to label the problem number clearly. There are five further problems that cover a wider range of material. In all problems be careful to label the variables and write units.

I: Required Questions:

1. (20 points) I went to Pompeii and bought the same number of salads and small pizzas. Salads cost two dollars each and pizzas cost six dollars each. I spent \$40 all together. Assuming that the equation $2S + 6P = 40$ is correct. Then

$$2S + 6P = 40.$$

Since $S = P$, I can write

$$2P + 6P = 40.$$

So

$$8P = 40.$$

The last equation says 8 pizzas is equal to \$40 so each pizza costs \$5.

What is wrong with the above reasoning? Be as detailed as possible. How would you try to help a student who made this mistake.

2. (20 points) A student writes: $(x + 1)^2 = x^2 + 1$. List at least three strategies for helping the student understand how to solve this problem. Take several sentences each to explain two of these. Consider in particular how to help a 'visual learner'.

3. (20 points) In explaining how to solve $6x = 4x + 18$, a teacher writes the following on the board.

Solution:

$$6x = 4x + 18$$

$$-4x \quad -4x$$

$$6x - 4x = 4x + 18 - 4x$$

$$2x = 18$$

$$/2 \quad /2$$

$$x = 9$$

Comment and suggest improvements.

4. (20 points) Explain the three uses of the ‘minus’ sign. In the process you should demonstrate you know the difference between a unary and a binary function.

5. (20 points) Joe walks from Jackson south to Roosevelt Road on Halsted. At the same time, Hank walks from Roosevelt north to Jackson on the same sidewalk. Roosevelt is 8 blocks south of Jackson. Hank walks 3 blocks north per minute; Joe walks 2 blocks south per minute. Draw a graph that represents both of their trips. Carefully label the axes and explain what system of coordinates you are using. Mark on your graph where the two meet and tell when and where this happens.

II. Open-ended Questions:

Answer as many of the following questions as you have time for. This is a chance to show what you know about mathematics and mathematical pedagogy.

6. (15 points) In an aquarium with 200 fish 99% of the fish are guppies. How many guppies do you have to remove so that after you have removed them 98% of the fish are guppies. If a student is surprised by this answer what would you say?

7. (20 points) A slow moving tractor-trailer moves along the Dan Ryan. It moves so slowly that a boy on the standing above the expressway estimates it's length as follows. First he walks the same direction of the truck and finds that 140 paces after beginning at the back of the truck he reaches the front. He turns and walks back. He reaches the back of the truck in 20 paces. How long is the truck? Give a formal algebraic solution or an intuitive explanation or both.

8. (15 points) Solve the following two problems.

- a. Which is larger, $\frac{12121}{12122}$ or $\frac{12121212121}{12121212122}$? Why?
- b. Which is larger, $\frac{12121212121}{12121212122}$ or $\frac{12121212121212121}{121212121212122}$? Why?

Give a general explanation of what is going on.

9. (20 points) Find all k so that $2 \cdot 10^k \equiv 5 \pmod{13}$. The solution is an arithmetical sequence. What does this mean? Give another example that is more likely to arise in 9th grade algebra (or prealgebra).

10. (15 points) The following prompt was given as part of a ‘writing in algebra class’. A local mathematics contest requires that student compete in mental computation. This means they are not allowed to use paper and pencil or calculators to find the answer; they must do everything mentally. Two students were given the following problem to test ‘order of operations’. In less than 5 seconds one student gave the correct answer. When asked how he could get the answer so fast, he replied, ‘I used the property of zero.’ Explain the property of zero and how this property helped the student answer the problem so quickly.

$$[5 + 7 \times 13 - 6] \div 15 \times (36 - 12 \times 3).$$

- a. Write a short answer to the prompt.
- b. Only about 10% of Algebra I and II students gave a correct answer to this question. Can you suggest a revision to the prompt that might produce a better result? What do you think is going on here?

11. Prove or disprove each of the following, where m is a natural number. If one of the equivalences fail, is there an implication which is true?

- a. ‘ m is even’ is equivalent to ‘ m^2 is even’.
- b. ‘ m is divisible by 4’ is equivalent to ‘ m^2 is divisible by 4’.