Final Exam: MTHT 400
Methods of Teaching Secondary Mathematics
December 7, 2004

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NAME:

I: Required Question:

1. (20 points) (This problem is taken from an 11th grade book.) Chips for Nuts, the giant computer store, sells printer cables that come equipped with connectors at each end. A ten foot cable with connectors costs \$13 and a six foot cable with connectors costs \$9. How much does an a eight foot cable cost. Set up and solve this problem for an algebra class. Be sure to identify all variables and units.

Do this problem on this page and turn it in by 6:00. You will be given a correct solution and a copy of your initial solution to critique in the last half of the exam.

Do as many of the remaining questions as you can.

2. (20 points) A student evaluates the expression $\sqrt{}^3(x^3 + y^3)$ as $= x + y$. List at least three strategies for helping the student understand how to do this problem. Take several sentences each to explain two of these. In particular, give a geometrical interpretation of 'cubing' that will help the student understand.

3. (20 points) What is the difference between an expression and an equation? How would you react if a student gave the following answer to the question, 'Write an expression for the area of a rectangle.'

$$2x + 2y = 0$$

4. (30 points) A high school class conducts the following experiment. They weigh a number of pieces of fruit (some grapes, apples, grapefruit, and pumpkins); they also measure the circumference of each piece of fruit. After making a table of circumference versus weight, they use the cubic regression program on the calculator to give them a cubic function for weight in terms of circumference.

Why is it reasonable to expect this relationship to be cubic? What is a plausible interpretation of the coefficient of the cubic term? Give arguments for or against this being a good example of a mathematical model. (I can be convinced either way.)

One time this experiment was conducted only with pumpkins and a student (well actually it was a teacher) noticed that there is a large coefficient for the quadratic term. She gave a geometric explanation for this. What was it?

5. (20 points) A primary teacher says 'you cannot subtract a smaller natural number from a larger one and get a natural number answer.' (Of course, the 'natural' is suppressed since that is the only kind of number primary students know.) Although this statement is correct, putting too much emphasis on it interferes with students later understanding of negative numbers. Explain how a 9th grade algebra teacher emphasizing 'get x by itself', makes the same kind of mistake.

6. (20 points) We discussed a number of ways of introducing and working with negative numbers in this class (number line, algebra project, hot and cold cubes, properties of fields ...). Which do you think will work best with ordinary 9th grade students and why?

7. (20 points) (taken from an 11th grade book) Find a graphing window which shows the eventual behavior and all turning points of the function:

$$y = .2x^3 - 5x^2 + 38 * x - 97.$$

What is the appropriate window? Sketch what you see.

How do you know your answer is correct? That is, what properties of a cubic polynomial guarantee that you have found all turning points and determined the eventual behavior. Give your answer as responses to the following two questions.

a. List the properties you would expect 11 th graders to give as a reason.

b. Briefly sketch (no more that a sentence or two) why, using calculus, you know the properties in the previous part correctly describe the phenomena.

8. (20 points) Explain the precept: ”concept before name” and give an example showing how this adage could influence teaching an algebra class.

9. (20 points) Here is a problem and part of a solution with the reasons as written in blue in the algebra text.

Solve $\log 5x + \log(x-1) = 2$ for x.

$$\log 5x + \log(x-1) = 2 \qquad \text{Write original equation.}$$

$$\log(5x(x-1)) = 2 \qquad \text{product property of logarithms.}$$

$$10^{\log(5x(x-1))} = 10^2 \qquad \text{exponentiating each side using base 10.}$$

$$5x^2 - 5x = 100 \qquad 10^{\log x} = x$$

$$x^2 - 5x = 20 \qquad \text{Write in standard form.}$$

$$(x-5)(x+4) = 0 \qquad \text{Factor.}$$

So the solution is $x = 5$ or $x = -4$.

Is this answer correct? If not first give a clear demonstration that the answer is wrong. Then identify the step in the solution that is incorrect and explain why. (Hint: I see this as a place where the word domain has a real use in the llth grade.)

Finally, do you think there are any ways in which the ‘reasons’ for the various steps could be improved.

10. The following problems are taken from high school algebra tests. In each case, if possible, indicate which answer is correct. Then evaluate the question. Be sure to explain why you do or do not like a particular question.

- 1. A student tries to solve the problem: How long does it take a car to travel 240 miles if the car’s rate is 60 mi/h? The student uses unit analysis to check the problem. Which set of units is correct.

- (A) $\frac{mi/h}{h}$

- (B) $\frac{h}{mi/h}$

- (C) $\frac{mi/h}{m}$

- (D) $\frac{mi}{mi/h}$

- 2. -(3-x) + 8 = 2x
- (A) 11
- (B) 5
- (C) $\frac{5}{3}$
- (D) none of the above
- 3. If $0 < x < 1$, which expression has the greatest value?
- (A) $x - 2$
- (B) $2x$
- (C) $2/x$
- (D) x^2
- 4. In 1981 the pollution in a local lake was rated at 1.4 parts per million. In 1985 the pollution was rated at 3.0 points per million. Which of the following expresses the rate of change in parts per million from 1981 to 1985?
- (A) $\frac{2}{5}$
- (B) $\frac{8}{5}$
- (C) $\frac{5}{8}$
- (D) $\frac{1}{4}$

A Solution to problem 1.

Let U be the cost in dollars of one foot of cable. Let V be the cost in dollars of one connector. Then we have two equations.

$$10U + 2V = 13$$

$$6U + 2V = 9$$

Subtracting the first equation from the second we get

$$4U = 4.$$

So $U = 1$ and substituting in the second equation

$$6 + 2V = 9$$

and $V = 1.5$.
So an eight foot cable costs $8 * 1 + 2(1.5) = 11$ dollars.

Question 11. (20 points)

If you got the problem wrong, examine your work and describe what you would tell a student who made that mistake.

If you got the problem correct but with a different method, briefly explain the connection between the two methods. Then discuss some problems you might expect an 11th grader to have with this problem.