Integers and polynomials
Homework due Sept. 28.

So many of the answers to the homework on integers and polynomials were nonresponsive to my intention that I can only conclude the questions were badly posed. I will return the papers with comments but not give grades on that assignment; I will give grades only on the following replacement. You are welcome to include your first paper next week to answer some the questions. You need to say. My response to question X on the first assingment answers the current question Y.

1. Explain in a couple of sentences the connection between high school algorithms for the *multiplication of binomials* (not the general ring axioms) and multiplication of integers.

   List five mental arithmetic problems that illustrate the connections between multiplication of binomials and multiplication of integers.

2. Look up the properties of the real numbers in an algebra text. Give the name of the algebra text. Compare them with the axioms for rings as given in class or on the web site. Does the algebra book mention anything about closure? If the one you choose first doesn't, talk to your classmates to find an algebra test that has a closure condition. Does it make sense?

3. Mersenne primes: In these two factoring problems, write a couple of sentences showing how would explain this process to a high school class.

   (a) Prove $X^n - 1$ factors if n is composite.

   (b) Explain why this shows that if $b = 2^j - 1$ and j is a composite number greater than 2, then b is composite.

4. Fermat primes:

   (a) Prove $X^n + 1$ factors if n is a composite number with an odd factor. Explain why the odd factor has to be there. Write two problems you would use in 11th grade to illustrate this.

   (b) Explain why this shows that if $b = 2^j - 1$ is prime then j must be a power of two.

5. Explain why even though there are infinitely many prime numbers there may still be only finitely many Mersenne and Fermat primes. Explain for an 11th grade class the status of Mersenne and Fermat primes. (There is a good chance this will be in high school text books in the next 10 years.)

6. Use the information in this assignment to sketch a response to the plaint of a high school student, 'What does this abstract stuff have to do with anything?' You may want to check the internet for topics such as: cryptography, RSA, Mersenne primes, Sarah Flannery (look at several of the links). I don't want a long response here - just of a couple of sentences.