The fundamental theorem of algebra

Homework due September 21.

1. Recall that the complex conjugate $\overline{u}$ of a complex number $u = a + bi$ is the complex number $a - bi$. Show that complex conjugation: $(u \mapsto \overline{u})$ satisfies: $\overline{u + v} = \overline{u} + \overline{v}$ and $\overline{u \cdot v} = \overline{u} \cdot \overline{v}$.)

2. Use the fact you have proved in 1 to show that if $f \in R[X]$ and $f(u) = 0$ then $f(\overline{u}) = 0$.

3. Assume the fundamental theorem of algebra. Prove that any polynomial in $R[X]$ can be factored in $R[X]$ into a product of linear and quadratic polynomials.

4. Explain the relation between the fundamental theorem of Algebra and the fundamental theorem of Arithmetic. Does this change your understanding of what factoring is about? How does this relationship influence the teaching of 'factoring' in high school?

5. Show that any polynomial of odd degree with real coefficients has at least one real root. a) Give an intutive explanation suitable for 12th grade students. (Hint: consider the graph.) b) give a proof suitable for MTHT 430.