

Trigonometry

Scoring: 3 points for each part of 1), 2 points for each of 3,4,5.

1. Let α be an angle inscribed in a unit circle. Let β be the central angle cutting off the same chord.
 - (a) Show $\beta = 2\alpha$ using the basic properties of similar triangles and congruence axioms.
 - (b) Show that $\sin(\alpha)$ is $1/2$ the length of the chord cut off by α .
 - (c) Deduce the law of sines. Explain the proof with a picture and complete description when α is an obtuse angle.

Answer: a) As I mentioned in class, a) is Proposition 20 book 3 of Euclid; he gives several cases but if you draw a line from the vertex through the center and not worry about it being perpendicular to the chord then there is one proof for all cases. (This is what Julienne did in class.)

a) implies that two acute inscribed angles (or two obtuse) inscribed angles which cut off the same chord are equal (as Euclid would say) or have the same measure (as modern textbooks say).

b) Once a) is established, we can move the vertex of α so that the line from the vertex through the center is perpendicular to the chord. With this picture, similar triangles quickly give the result.

I charged one point if there was no indication that you were making a particular choice so that the line from the vertex through the center is perpendicular to the chord.

c) I won't draw it out here but the intention was to take the proof from the website (<http://www.cut-the-knot.org/proofs/ptolemy.shtml>) and see that it works just as well for an obtuse angle in showing that the ratio of opposite side to sin is the same for all angles of a triangle embedded in a circle.

2. Explain the difference between right angle and unit circle trigonometry.

Answer: Right angle trigonometry is about similarity; it deals only with angles of at most 90° . Unit circle trigonometry adds the concept of periodicity. Angles of any size are studied. The notion of similarity is much more basic so the concepts of right angle trigonometry can be introduced at a much earlier age (sometimes in junior high). Doing so allows students to focus on periodicity when they study unit circle trigonometry.

Note that similarity is also key for unit circle trigonometry. While the sin of an angle is the y -coordinate of the point where the ray with that angle to the x -axis crosses the unit circle, it is equally the ratio of the y -coordinate of the point where the ray with that angle to the x -axis crosses

the circle of radius r to r . Pictures illustrating this and even experiments with concentric circles are excellent activities for introducing unit circle trigonometry.

Thus \sin is always dimensionless.

3. What is an algorithm? (one or two sentences) What are the characteristics of a good description of an algorithm? (at most 3 or 4 sentences).

Answer: An algorithm is an effective procedure for finding the solution to a class of problems. A good description of an algorithm makes clear what the input and output are and lists the steps of the procedure precisely.

4. Why do I rail against ‘foil’ and have no complaint about ‘sahcahtoha’? (Whoops, I had to look it up on the internet sohcahtoa.) (This isn’t just psychology; there is an important mathematical difference.)

FOIL is a mnemonic that transfers a process about which student should think (the use of the distributive law) into a mindless routine. It often admits false generalizations.

SOHCAHTOA is a mnemonic to remind people of a *convention* that must be memorized and cannot be thought about. There would be no difference in mathematics at all if \sin were adjacent over hypotenuse and cosine opposite over hypotenuse. Someone made an arbitrary choice of the name and we are stuck with memorizing it. (There is also a risk of overgeneralization here as someone may apply the mnemonic to non-right triangles.)