When are equations equivalent?

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September 20, 2004

The main problem here is taken from an article in the September 2004 issue of the News Bulletin of the NCTM (based on ideas of Joanne Canigila of Eastern Michigan University).

Are you sure about that?

As tall as Yao? Many people wish they could be a tall as the basketball player Yao Ming. The 7' 6" Ming plays for the Houston Rockets in the NBA and has appeared in television commercials.

Consider the following argument.

Let H be your height, Y be Yao's height and A be the average of the two heights, $(H + Y)/2$. Then $H + Y = 2A$ so that:

$$(H + Y)(H - Y) = 2A(H - Y) \text{ or}$$

$$H^2 - Y^2 = 2AH - 2AY \text{ or}$$

$$H^2 - 2AH = Y^2 - 2AY \text{ or, adding } A^2 \text{ to both sides of the equation}$$

$$H^2 - 2AH + A^2 = Y^2 - 2AY + A^2$$

$$(H - A)^2 = (Y - A)^2.$$

Thus, $H - A = Y - A$ or $H = Y$. You are indeed as tall as Yao! If you do not think you can ever be as tall as Yao, find the flaw in this argument.

Assignment

1. First we study the problem as stated.

   (a) Explain the flaw in the argument.

   (b) Demonstrate the flaw in the argument using the graph of an appropriate function.

2. Now we analyze some of the fine points of the presentation.

   (a) Explain the relationship between the equations in the argument presented above. If you use a technical word like 'equivalent', explain in the language of 9th graders what it means.

   (b) The author used the word 'or' three times in stating the problem. What meaning was intended for 'or' in this context. Is that a good word to use for high school students? What are some alternatives?

   (c) How do you justify each step in argument above. That is, why are all (well, almost all) of the equations 'equivalent'.

   (d) In fact, where I wrote 'H be your height, the author wrote '$H = $ your height'. Can you explain why I made this change?

3. Feedback: For each of the first four problems in this course: water and wine, walking on Halsted, a strange number, and 'As tall as Yao', do you thing the mathematics was too easy, about right, or too hard? Did each of them teach you something about teaching? (yes or no for each problem). Do you have further comments on the homework?