Mersenne and Fermat primes

John T. Baldwin

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Notice that if j is even, $2^j - 1$ is composite unless $j = 2$. We want to find many generalizations of this fact. (Recall that a number is composite if it has a factor other than itself and 1.)

1. Consider numbers of the form $b = 2^j - 1$. A prime of this form is called a Mersenne prime.
   (a) Try to give a property of j (even, odd, prime, composite, etc) that guarantees b is prime or a different property that guarantees that b is composite.
   (b) Are there infinitely many primes of the form $b = 2^j - 1$. Yes, no, I don't know.
2. Consider numbers of the form $c = 2^j + 1$.
   (a) Try to give a property of j (even, odd, prime, composite, etc) that guarantees c is prime or a different property that guarantees that c is composite.
   (b) Are there infinitely many primes of the form $c = 2^j + 1$. Yes, no, I don't know.
   (c) Fermat primes are of a bit more special form than $c = 2^j + 1$. On the basis of answering the earlier questions you should be able to explain the terminology; do so.

This problem is open-ended in the sense that you have a lot of choices of what properties to choose and you have to decide whether you can 'make' the number prime or 'make' the number composite. But for this course a full solution of the problem will involve showing patterns of factoring that are a little more complex than usually taught is high school but readily accessible to high school seniors. A really full solution would make you famous!