Modeling

A. Consider the following problems involving modeling data by a polynomial equation. In each case, give a name for each parameter, give the units of the parameters, and write a sentence or two to give a plausibility argument for the model. (I copied these directly from books except when the problem was so ill-posed I couldn't bear to copy it.)

1. The number of calories C that a person burns performing an activity varies directly with the number of minutes t that the person does the activity. A 160 pound person burns 73 calories by dancing for 20 minutes. Write a linear model that gives C as a function of t.

2. When the length ℓ of a rectangle is fixed, the area (in square inches) of a rectangle varies directly with its width w in inches. When the width of a rectangle with length ℓ is 12 inches, its area is 36 square inches. Write an equation that gives A as a function of w. (This particular kind of problem is particularly hard to state correctly and precisely. I cleaned up a version that didn't make any literal sense. I am not happy with the result. Write this problem correctly.)

3. Here is data concerning the price of 14 caret gold chains.

length x in inches	16	18	20	24	30
price c in dollars	308	344	380	452	560

   If it makes sense, write an equation giving c as a function of x.

4. On any planet the height h (in feet) of a body t seconds after is dropped can be modeled by $h = \frac{-gt^2}{2} + h_0$. A rock dropped from a height of 200 feet takes 5 seconds to reach the ground on earth but just under $\sqrt{5}$ seconds on Jupiter. Explain this difference. (Hint: What, exactly are g and h_0?)

Be sure that you identified the parameters in each part of A.

B. Suppose f is a function from the real numbers to themselves. Prove:

1. If f is a linear function then f has the property

   (*): There is a real number m such that for all real x, $f(x+1) - f(x) = m$.

2. If f has the property (*) then the function defined for all real x by $L(x) = mx + f(0)$ agrees with f at all nonnegative integers.

Please turn over.

C. Consider the data:

x	2	3	4	5	6
$f(x)$	5.53	7.68	9.83	11.98	14.13

By considering $f(x+1) - f(x)$ find a linear function $L(x)$ which has the same values on the given points as f. Let $h(x) = L(x) + \sin(2\pi x)$. Show that h agrees with $L(x)$ on all integers but is not a linear function. Sketch the graph of $h(x)$.

D. The data in the table below give the average speed y in knots of the trident motor yacht for several different engine speeds x (in hundreds of revolutions per minute RPM).

engine speed x	9	11	13	15	17	19	21.5
boat speed y	6.43	7.61	8.82	9.86	10.88	12.36	15.24

An algebra text asserts that this data can be modeled by the polynomial equation:

$$y = .475x^3 - 0.194x^2 + 3.13x - 9.53.$$

This result was obtained by asking the TI-83 to do a cubic regression. I asked it to do a quartic regression on the same data and got

$$y = 4.311x^4 - .0215x^3 + .387x^2 - 2.405x + 9.557.$$

Can you think of any reason to prefer one model to the other? Can you give any physical meaning to the parameters in either case?

With what degree of polynomial can you guarantee a better fit. (I am not asking you to find the coefficients, just tell me the theoretical result.)

E. Look at a few algebra texts or 'teaching tips' articles and find the silliest example you can of a book giving a mathematical model for what is in fact random data.