rubric: Extensions to: Strange Number

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If you want to improve the grade on your assignment you must redo the work correcting the difficulty and answer the two questions posed below. Students with scores below 5 can do the make-up; the maximum score for the repeated work will be 6.

There is an abbreviated version of the assignment below. Problem 1 was worth 2 points, problem 2, 3 points and problem 3, 3 points. The first two were graded on the quality of explanation since I had told you the answer. For the third, I gave up to 3 points for naming at least three of the following possible correct answers to 3a).

1. multiplication tables and the multiplication algorithm
2. place value
3. how to set up equations
4. how to solve equations
5. An equation may have more solutions than the 'real' problem it was set up to solve
6. periodicity and the rational numbers; repeating fractions
7. modular arithmetic
8. manipulating exponents

I did not assign points for 3b) because I saw no point in either giving points for people who can tell me back my views or penalizing those who disagree on what is in some ways a matter of opinion. Nevertheless, *the correct answer* is: If you do not teach mathematical problems that have internal motivation you are depriving your students of the most important part of the subject. You are preparing to be teachers of mathematics, not engineering or social science. (Maybe another time, I will polish this and phrase it positively.)

Thus the total possible for the assignment was 8 points. In addition I gave up to 5 bonus points for those who explored the wonders of weird numbers.

1. $3^{k+1} \equiv 1 \mod 7$.)

What are all the possible choices of k to satisfy this equation and thus answer the original problem.

2. Playing around with $\frac{2}{13} = .\overline{153846}$, I found a solution to the puzzle: Find a 5 digit number such that if put a 1 after the number and multiply by 2 I get a number 7 times as large as the one obtained by putting a 1 in front of the number.

Like problem 1, show that if I replace 5 by k, I get that the possible k are all numbers of the form $5 + 6r$ for any positive integer r.

3. Write a short note discussing a) what mathematical ideas are illuminated by different solutions to the strange number problem and b) the idea that mathematics can be motivated either externally (finding out how long a trip takes) or internally (solving questions that arise within mathematics). Should both of these motivations play a role in high school teaching?

4. The last part is completely open-ended. Try to work with various repeating decimals to find further riddles of this sort. If you find a method for transforming fractions m/n to problems this is great but I haven't gotten that far. This is just supposed to be 'fun'. See what you can do.

Make up questions

1. Explain the difference between the unary operator '$5 \bmod 7$ and the binary equivalence relation $x \equiv y \bmod 7$. To avoid confusion, the first usage is sometimes written %. I have never seen mod as a unary operator in a math book but students use it all the time.

2. Make up and solve completely one new strange number problem. This means make up a word problem, solve it algebraically, then check for which values of k, the problem can be solved.