Weinzweig's axioms
(corrected Jan. 15; extended Feb. 17)

Definition 1 *A* geometry *is a triple* $\langle \Pi, \mathcal{L}, \Psi \rangle$ *(points, lines, halfplanes) where each* $\ell \in \mathcal{L}$ *is a subset of* Π *and* Ψ *assigns to each* ℓ *a pair of subsets* Ψ_ℓ^1, Ψ_ℓ^2 *of* Π *- the halfplanes associated with* ℓ*.*

Axiom 2 (I) *Two points determine a line.*

Axiom 3 (II) Ψ_ℓ^1, Ψ_ℓ^2 *and* ℓ *are a partition of* Π*; neither halfplane is empty.*

Definition 4 $(A - \ell - B)$ *The line* ℓ separates *the points* A *and* B *if* A *and* B *are in different halfplanes of* ℓ*.*

[$A - C - B$*] The point* C separates *the points* A *and* B *if* C *is on a line* ℓ *that separates* A *and* B *and* C *is on* AB*.*

[$a - \ell - b$*] The line* ℓ separates *the lines* a *and* b *if* a *and* b *are subsets of different halfplanes of* ℓ*.*

Axiom 5 (III) *If* ℓ *separates the points* A *and* B *then* ℓ *meets* AB *at some* C*.*

If C *separates* A *and* B *and* m *is any line that intersects* AB *at* C *then* m *separates* A *and* B*.*

Definition 6

1. *The* closed half plane $\overline{\Psi_\ell}$ *is* $\Psi_\ell \cup \ell$*.*
2. *If* $\ell \cap AB = A$*,* ψ_B^ℓ *is the* ℓ*-half plane containing* B*. And the* ray $\overrightarrow{AB} = \overline{\psi_B^\ell} \cap \overline{AB}$
3. *For distinct points* A, B*, the em segment between* A *and* B*,* $\overleftrightarrow{AB}$ *is* $\overrightarrow{AB} \cap \overrightarrow{BA}$*.*
4. *The* open segment *or* interior *of* $\overleftrightarrow{AB}$ *is denoted* ${}^o\overline{AB}^o$*:*

$$ {}^o\overline{AB}^o = \overleftrightarrow{AB} - \{A, B\} $$

Axiom 7 (IV) C *separates* A *and* B *if and only if* $C \in {}^o\overline{AB}^o$*.*

Axiom 8 (V) *There are two points.*

Definition 9 *There is one more component to a geometry. There is a function* Γ *which assigns to each line a mapping from* Π *into* Π*.*

We write Γ_ℓ for the function assigned to ℓ. We denote the composition of two such functions by $\Gamma_\ell \Gamma_m$ and write 1 for the identity map on Π.

Recall that a mapping f is said to fix a point A if $f(A) = A$. A mapping f is said to fix a set X setwise if for every $A \in X$, $f(A) \in X$.

Axiom 10 (VI) *For every ℓ, Γ_ℓ fixes each element of ℓ. For every $A \notin \ell$,*

$$A - \ell - -\Gamma_(\ell)\{A\}.$$

Axiom 11 (VII) *For every ℓ, $\Gamma_\ell \Gamma_\ell = 1$.*

Axiom 12 (VIII) *For every ℓ, Γ_ℓ preserves rays.*

Definition 13 *The group of mapping generated by the Γ_ℓ is called the group of rigid motions.*

Axiom 14 (IX) *For every ray $\vec{AB}$, if a motion fixes a ray set-wise then it is either the identity or Γ_{AB}.*

Definition 15 *a is perpendicular to b if Γ_b fixes a setwise.*

The following is a version of the parallel postulate.

Axiom 16 (X) *If $a \parallel b$ and $\ell \perp a$ then $\ell \perp b$.*

Definition 17 *The line t is the bisector of $\angle BAC$ if $\Gamma_t(AC) = AB$.*

Axiom 18 (XI) *There exists an angle bisector for every angle.*

I prefer the equivalent version of Axiom XI: If a and b are intersecting lines there is a reflection mapping a to b.