**MthT 411 Spring 2008 First Examination**
**March 6, 2008, 5:00–6:15**

We are not trying to find the minimum axiom set for each result. But, for each problem, state the highest number of an axiom used in your proof, directly or indirectly.

Here are possible solutions. Others were given by various students. I give some notes on the grading. I took points off for false statements and if there were a host of irrelevant statements that made it difficult to follow the main argument. But most deductions are for explicit omissions mentioned below.

1. Prove that if $l$ is a line and $A, B$ are points, then

$$A - l - B \iff l \cdot {}^{\circ}\overline{AB}^{\circ} \neq \emptyset.$$

   Proof.

   Suppose $l \cdot {}^{\circ}\overline{AB}^{\circ} \neq \emptyset$; say $AB \cdot l = C$. By Axiom IV, $A - C - B$ and so by Definition $A - l - B$.

   Now, suppose $A - l - B$. By definition, there is a $C \in l$ with $A - C - B$. By Axiom IV again, $C \in {}^{\circ}\overline{AB}^{\circ}$.

   **There were 16 points for each direction. Each required Axiom IV and failure to mention axiom 4 cost one half of that direction.**

2. Prove that if $l$ is a line, then $\Gamma_l[\Psi_l^1] = \Psi_l^2$.

   Proof. For any $P \in \Psi_l^1$, Axiom VI asserts $\Gamma_l(P) \in \Psi_l^2$. Moreover, 9.13 shows $\Gamma_l$ is onto and Axiom VI says no element of $\overline{\Psi_l^2}$ is in $\Gamma_l(\Psi_l^1)$. So $\Gamma_l$ maps $[\Psi_l^1]$ onto $\Psi_l^2$. This uses results up to Axiom VI.

   **It is essential to give an argument that $\Gamma_l$ maps *onto* $\Psi_l^2$. I used a direct argument above; many students correctly quoted 9.43. Failure to mention this cost 8 point but I didn't deduct in both parts 1) and 2) since it is the same mistake.**

3. Prove that every triangle has infinitely many interior points.

   Proof. Let $\triangle ABC$ be a triangle. By Theorem 8.45 there exists $D \in {}^{\circ}\overline{AB}^{\circ}$ and $E \in {}^{\circ}\overline{BC}^{\circ}$. By Theorem 8.33, the segment ${}^{\circ}\overline{DE}^{\circ}$ lies in the interior of $\triangle ABC$. But by Theorem 8.50, ${}^{\circ}\overline{DE}^{\circ}$ is infinite. I am quoting results proved after Axiom V.

   **I deducted 10 points for just saying one can continue a construction to get infinitely many points.**

**grades 80 up A; 65-80 B ; 50- 65 C**