Math 215 - Assignment Number 7, Spring 2013

Read Chapters 10,11,12,13,14. Read the notes on sets by Rudin that we have on our website. Read also the supplementary notes on infinite sets from the website and the supplementary notes on the Cantor-Schroeder-Bernstein Theorem.

I. Eccles page 182. Problem Numbers 4, 5, 11, 14, 16, 17, 24.

II. Let

$$S_k(n) = 1^k + 2^k + \cdots + (n-1)^k.$$

(a) Let l be a positive integer, and show that

$$(l+1)^{k+1} - l^{k+1} = 1 + C_1^{k+1} l + C_2^{k+1} l^2 + C_3^{k+1} l^3 + \cdots + C_k^{k+1} l^k.$$

(b) Show that

$$n^{k+1} = \sum_{l=0}^{n-1} [(l+1)^{k+1} - l^{k+1}]$$

for positive integers n.

(c) Use parts (a) and (b) to prove that

$$n^{k+1} = n + C_1^{k+1} S_1(n) + C_2^{k+1} S_2(n) + \cdots + C_{k-1}^{k+1} S_{k-1}(n) + C_k^{k+1} S_k(n).$$

(d) Use part (c) to obtain explicit formulas for $S_k(n)$ for $k = 1, 2, 3, 4$. Compare your results with the already known formulas

1. $S_1(n) = n(n-1)/2$.
2. $S_2(n) = (2n^3 - 3n^2 + n)/6$.
3. $S_3(n) = (n-1)^2 n^2/4$.

III. Use the notes on the Cantor-Schroeder-Bernstein Theorem and give an explicit one-to-one correspondence between the following pair of sets.

1. $A =$ the set of real numbers that are greater than zero.
2. $B =$ the set of real numbers that are greater than or equal to one.