CHAPTER TWO

Must mathematical physics be reductionist?

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I think that one of the things to be learned from Freeman Dyson's presentation is that one cannot be a self-respecting scientist without also being a rebel. This would seem to imply that in this book we are urged to rebel against the point of view of reductionism. This is difficult to do unless we know first what reductionism is; and in fact I am not clear that I do know what the word means. Some forms of reductionism are almost synonymous with 'scientific', in which case we find ourselves driven to a contradiction. If we are going to be scientific, we have to be rebellious, but that rebelliousness leads us to resist the critical stance of the symposium—which would mean science itself if we take 'reductionist' to mean merely 'scientific'.

I am going to interpret the word 'reductionism' in various ways that are different from this. I have certainly been accused of being non-reductionist, and I have to try to understand what this charge means. There are perhaps two main alternative interpretations that come to mind, and I shall try to illustrate them in this contribution.

There are indeed certain areas of mathematics that are, in one real sense, non-reductionist. This does not prevent us from discussing problems in those areas in a very clear and characteristically mathematical way. We might take reductionism to mean the breaking-down of things into smaller and smaller parts, so that if you understand how the small parts work, that will in principle tell you how the big thing works. It may be that you need other kinds of ideas to say interesting things about the big things, as Freeman Dyson stressed, but here, the idea of reductionism is that the behaviour of the big things we study is governed by the behaviour of the individual units of which they are composed.

That is one strand of reductionism. Another strand of reductionism may be related to this by asking if knowledge of the present behaviour of those small units will allow us to predict the behaviour of the big things in the future? Determinism has been a central concern of philosophy and physics for many centuries: we may understand it as the proposition that the behaviour of the system at one time will determine what it does at a later time. It is important to emphasize that the *computability* of the behaviour at later times is a separate question. I want to stress the difference between computability and determinism because the distinction may prove to be an important one. Before we address it, I should like to return to our first interpretation of reductionism and illustrate how it is possible to talk about holistic concepts in a clear and mathematically precise fashion.

HOLISM AS MATHEMATICS

Let us start by investigating what we mean by a holistic concept. Figure 2.1 illustrates an impossible triangle. By virtue of what particular property is the triangle impossible? The fact that it possesses this feature of impossibility is clear. It is supposed to be an image that conveys to the mind a particular three-dimensional object, but that three-dimensional object simply cannot exist in ordinary space. We must ask ourselves what is wrong with the picture: can we point to somewhere in the picture where the mistake was made?

We might locate the impossibility in one specified corner of the triangle, so that if we covered up that corner the figure would make sense as the

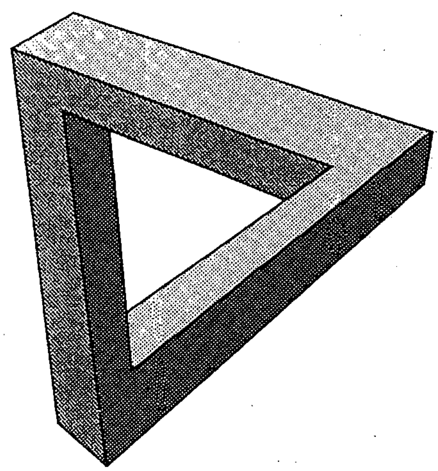

Fig. 2.1. An impossible triangle. Where is the impossibility?

representation of a possible three-dimensional object. We could then say that the impossibility has disappeared. Apparently it did not reside in the rest of the picture. If we hide *any* one corner, or remove *any* edge, the structure suddenly becomes possible. When we study the complete picture, however, the impossibility re-emerges.

This impossibility is a property of the whole structure. It cannot be localized in some part: it is a feature of the complete mathematical object, so it is a *holistic* property. There are areas of mathematics dedicated to discussing this kind of property in a rigorous way.

Let us imagine breaking the figure into three pieces, as illustrated (Fig. 2.2), and then glueing the parts together again. We could continue breaking the structure down, and each of the pieces would be a possible object in ordinary space. The process of glueing them together is the operation which eventually produces the impossible triangle. In technical mathematical terms, we take all the pieces together with all the specific glueing operations and by a factoring-out procedure, we extract what is called the cohomology. This mathematical concept abstracts what it is that interests us: the measure of impossibility of the triangle. In this particular instance, we can give this measure as a single real number, describing the 'degree of impossibility' of the triangle.

If I break this structure somewhere, so that it becomes a possible three-dimensional object, I can measure the ratio of the distances from my eye of one end of the break to the other end of the break. That ratio is a measure of the

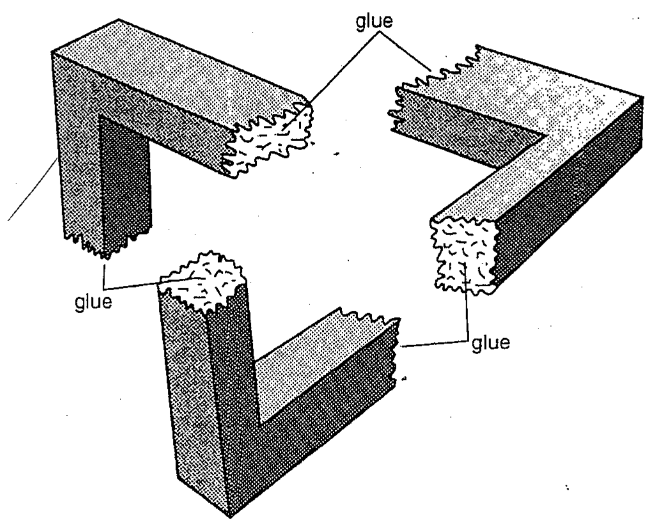

Fig. 2.2. Cohomology: an impossible triangle broken and reassembled.

impossibility of the structure and it will be the same, no matter where I cut the figure. This holds also with more than one break, no matter how many pieces I reduce it into, provided that I multiply all the relevant factors together. That perhaps gives you something of the idea of cohomology. In this particular structure, cohomology supplies the degree of impossibility of the object, and it is a perfectly well-defined and precise holistic mathematical concept.

If we take a knot (Fig. 2.3), we might ask another question: where does the property of knottedness reside in this knot? Like the impossibility of the triangle, we find that the 'knottedness' cannot be localized. It is a property of the structure as a whole. There are profound mathematical theories that deal with this question of knottedness: and though it looks simple enough, knottedness is more difficult to deal with than cohomology. Research in this field dates from the turn of the century and more recently and is now very well developed.

Another example is a Möbius band (Fig. 2.4). Where, we might ask, is the twist? If we take a piece of paper, twist it once and glue it together, we get a Möbius band. Then wherever we cut it, the twist is gone. It does not matter

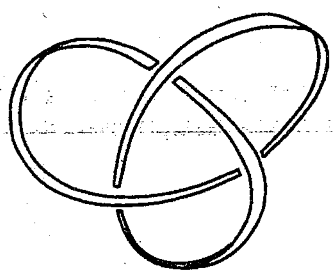

Fig. 2.3. Where does the knottedness reside in the knot?

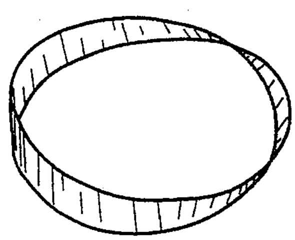

Fig. 2.4. A Möbius band. Where is the twist?

Then . . . "Then these gates are just packet switching cards in an old-fashioned wide-area network?"

Laing said, "Sort of."

Sort of. Pulse of fear. "Then how do we know . . ." He suddenly felt very short of breath. No way in. No way out. "How do we know there's any reality beyond this network?"

Laing, grinning, "We don't." Something mean-spirited in her eyes.

Kincaid said, "Some of us got in here from an external reality."

Ling said, "If that's all the gates are, then it must be an 'external reality' that's . . . part of this network." No. I do not wish to believe that I'm not real. I . . .

Genda said, "That's an easy inference to draw, professor. It'd simplify things no end. But the fact is, all quantum processes are linked through the Toolbox to Platonic Reality. It's why you have an experience you call consciousness."

Laing said, "And why I do, though I'm just a subroutine in a star-spanning information network."

Genda: "And so, when we go through the gate, we link to the other parts of the *real* gatenet, access the Multiverse, and what was real in here is also real out there."

An hour, then a day, another day then another, while *Baka-no-Koto* spun through the dark between the stars, that classic dark of song and story, the stars still no more than jewels against the sky, unmoving. Unlike, *most* unlike all the old images . . .

Robert Bruce Tanner Davidson the Third sat alone down in the hyperdrive bay, sitting stripped to a pair of black-and-gold paisley briefs, sweating into the drive bay's hot, dry air, air like the inside of a kiln, looking out through the hull at the motionless sky.

Sky with the power to entrance. The power to entrance me, magic me away from everything mundane, though I'll soon be two hundred years old . . .

He kept glancing back at the Type One hyperdrive, technically the Harveson Translight Overdrive Generator unit. That had the power to entrance as well. The power to call forth

memory of emotion, of remembered fancy. Little gears and spinning disks, disks that cast forth rainbows, disks that sang as they turned to mist. Little masts projecting here and there, turning spindles bound together by silvery bits of whirling Möbius strip.

More mist. A whiff of distant suns. Did Seacaptain Chandler tell us true? If I fall in there will I come back inside-out? Will my comrades have to shoot me out of my misery? Or will the ghostgirl do for me what she did for the German soldier's glove?

Too many close calls with death. A man my age should be long dead. Maybe I am dead. I don't even remember the Korean War, though I was already a school-age boy, though my unremembered father died in that faraway land.

Far away? He never knew. I never imagined. So many different lives. So many possible histories. If I hadn't studied aerospace engineering, I'd never have grown rich building rocket engine components for the American Renaissance. And if I hadn't grown rich, I wouldn't have had the medical care I needed, two heart transplants, an artificial liver, three new sets of kidneys as I grew older and older still.

I was 112 years old the day I sat with my shawl in my lap, on a balcony overlooking the garden of my country home, when that cute little nurse brought me the paper, *Times* was it? Little nursie shaking with excitement, and I read the headline, "Eternal Life."

I almost died laughing that day. Almost. She had to defib me twice before the medevac Moller came to cart me away for treatment . . .

Outside, the stars were ruby and amber, sapphire and bright, bright gold. The magic hatch to the drive bay dilated open and an immense, angular black shape wriggled in, filling the room with the sound of breath, the faint scent of living flesh. Tarantellula said, "Wondered how you managed to disappear."

Brucie Big-Dick looked at her and smiled: "I never really enjoyed crowds." She'd gotten out of her Marine combat fatigues, was wearing what looked like a pair of white silk briefs and some kind of thin halter-top. Black breasts con-

From "The Transmigration of Souls" by William Barton

From "Knots on a Torus — A not so Standard Model" by Jack Arvin

2. Geometrical Preliminaries.

2a. Genesis.

Before we describe the model in any detail, it may be revealing to recount an incident - - actually an epiphany of sorts - - that your intrepid author experienced, in fact, one that initiated the entire process of model development. In the waning years of the 20th century said author was staring at the cover of a small book published in 1964, coincidently a book on recreational topology [10]. Here's what the cover looks like (**Figure 2-1**); no printing, no information other than the pictured diagram, which clearly depicts a flattened Moebius strip, in fact, the canonical one half-twist MS, ubiquitous in elementary topological disquisition (and in commercial logos).

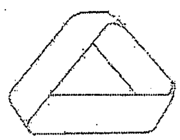

Figure 2-1

But now, suppose, as in **figure 2-2**, we arbitrarily associate a direction of ***traverse***, say counterclockwise, to such a diagram. Then we can characterize the corner folds as *two down into the plane* of the diagram *and one up out of it*. And if we start out with an MS with the ***same*** direction of traverse but with the ***opposite*** direction of twist, we end up with *one down and two up* folds.

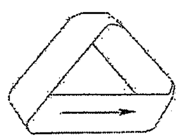

Figure 2-2

The astute reader will undoubtedly have seen where this is going[a]; by coincidence, 1964, was also the year that ***Gell-Mann*** and ***Zweig*** published their ground-breaking theory of quarks as the fractionally charged, component particles of the hadrons, *two ups and a down for the proton* and *two downs and an up for the neutron* with antiquarks making up the corresponding antiparticles and various combinations of a quark and an antiquark for mesons. For our intrepid author the connection was immediate (and undoubtedly so for the reader (s) of this paper as well) and soon led to the development of the alternative model,which we (finally!) summarize below.

[a] Not to imply that not seeing indicates non-astuteness.