M417

Fall 1996

hw14.tex due Nov 25, 1996

1. Find

$$\textbf{p.v.} \int_0^\infty \frac{t^{z-1}}{t-1}\, dt, 0 < \mathrm{Re} z < 1.$$

   Express your answer in terms of trigonometric functions (of πz).

2. Let C be a simple closed contour and C_i be the interior of C. Suppose that $f(z)$ is analytic and nonzero on C, meromorphic in C_i, and that in C_i, f has zeroes at $a_1, \ldots, a_N$, and poles at $b_1, \ldots, b_M$. Let $H(z)$ be analytic on C and C_i. Then

$$\frac{1}{2\pi i} \oint_C H(z) \frac{f'(z)}{f(z)}\, dz = \sum_{j=1}^{N} H(a_j) - \sum_{k=1}^{M} H(b_k),$$

   where each zero and pole occurs as often in the sum as is required by its multiplicity.