M417

Fall 1996

hw4.tex due September 25, 1996

1. Find
$$\frac{d\tan^{-1} z}{dz}$$

2.
   a) Find a [multivalued] function $F_+(z)$ such that $\dfrac{dF_+}{dz} = \dfrac{1}{1+iz}$.

   b) Find a [multivalued] function $F_-(z)$ such that $\dfrac{dF_-}{dz} = \dfrac{1}{1-iz}$.

3. Show that
$$\frac{1}{1+z^2} = \frac{1}{2}\left\{\frac{1}{1+iz} + \frac{1}{1-iz}\right\}.$$

4. Find a (single valued!) function $F(z)$ defined in the strip
$$\{-1 < \Im z < 1\}$$
such that
   - $F(z)$ is analytic (single valued!),
   - $\dfrac{dF}{dz} = \dfrac{1}{1+z^2}$,
   - $F(0) = \pi$.

   What is $F(1)$?