M417

Fall 1996

xfs.tex

1. Find the poles and zeroes, with multiplicities and residues of

$$f(z) = \tan(\pi z).$$

2. For a real, find

$$\lim_{y \to \pm\infty} \tan(\pi(a + iy))$$

3. For $0 < a < \frac{1}{2}$, what is the change of $\arg(\tan(\pi z))$ along the curve [line]

$$C_a = \{z(t) = a + it | -\infty \leq t \leq +\infty\}?$$

   Hint: What is the sign of the real part of $\tan(\pi z)$ along C_a?

4. For $\frac{1}{2} < a < 1$, what is the change of $\arg(\tan(\pi z))$ along the curve [line]

$$C_a = \{z(t) = a + it | -\infty \leq t \leq +\infty\}?$$

   Hint: How many zeroes and poles of $f(z)$ are there between $C_{\frac{1}{4}}$ and $C_{\frac{3}{4}}$?

5. For various values of z_0, find the radius of convergence of the Taylor series for $f(z)$ about $z = z_0$.

6. Discuss the [multiple valued] functions $\sqrt{z}$ and $z^{\sqrt{2}}$.