CRITICAL NUMBERS Solve $f'=0$
SPECIAL PTS
INC on interval if $f'>0$
DEC on interval if $f'<0$
INC/DEC can change only at $\begin{cases} \text{CRIT} \\ \text{SPECIAL} \neq 0, \ldots, \text{end} \end{cases}$
RELATIVE MIN/MAX

SECOND DERIV: f'' (computable)

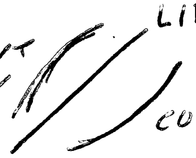
Defn. : CONCAVE UPWARD $f''>0$ Consistent

DOWNWARD

Picture Con

glue to y

THREE WAYS to BE INC

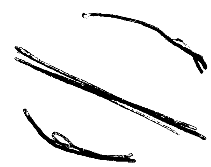


concave DOWN

LINE

CONCAVE UP

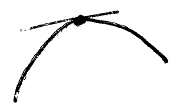
THREE WAYS TO DEC $f'<0$ $f''<0$

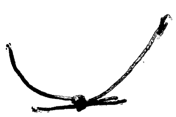


LINE $f'=$ const, $f''=0$

$f'<0$ $f'>0$

LOCAL MAX





$f'=0$, $f''>0$ Ecn LOCAL MIN

CONCAVITY CAN CHANGE ONLY AT $\begin{cases} f''=0 \\ \text{SPECIAL (and } f'' \text{ does not exist)} \end{cases}$

INFLECTION POINT

~~Example~~ 3.2.3
Det INC/DEC RELATIVE MAX/MIN ; INFLECTION

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$$y = 3x^4 - 2x^3 - 12x^2 + 18x + 13$$

SPECIAL NONE

$$\frac{dy}{dx} = 12x^3 - 6x^2 - 24x + 18 \quad (=0?$$

Get Lucky $x = 1$.

$$\cancel{x+1)} \quad x-1 \overline{) \cancel{3x^4 - 2x^3 - 12x^2 + 18x + 13}}$$

$$\cancel{12x^2 + 6x - 18}$$

$$x - 1 \overline{) 12x^3 - 6x^2 - 24x + 18}$$

$$\underline{12x^3 - 12x^2}$$

$$6x^2 - 24x + 18$$

$$\underline{6x^2 - 6x \cancel{+}}$$

$$-18x + 18$$

Lucky Again.

$$(x-1)^2 (12x^2 + 6x - 18)$$

$$= 6(x-1)(2x^2 + x - 3)$$

$$= 6(x-1)(2x+3)(x-1)$$

$\boxed{\text{Phew!!}}$

$$\frac{d^2y}{dx^2} = 36x^2 - 12x - 24 = 12(3x^2 - x - 2)$$

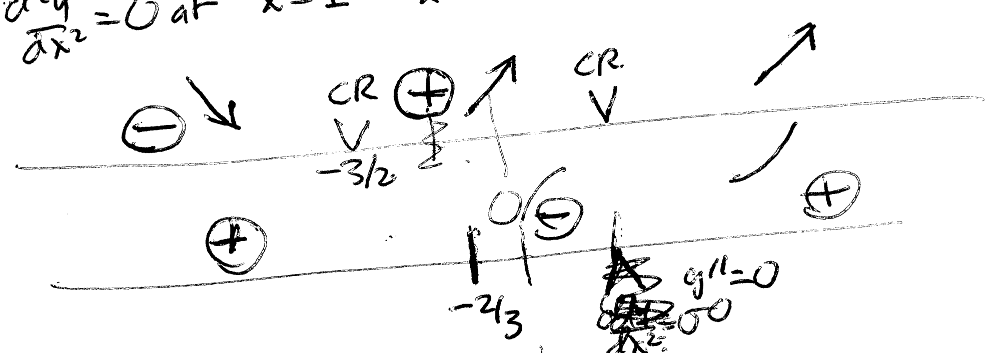
$$= \cancel{6(6x^2 - 2x} \qquad = 12(x-1)(3x+2)$$

$$= \cancel{0 \text{ at}}$$

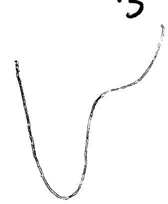
(That's the hard part.!)

CR. NOS. $x = 1, \ x = 1, \ x = -3/2$

$\frac{d^2y}{dx^2} = 0$ at $x = 1 \quad x = -2/3$



$\boxed{\text{Graph at CR NOS}}$

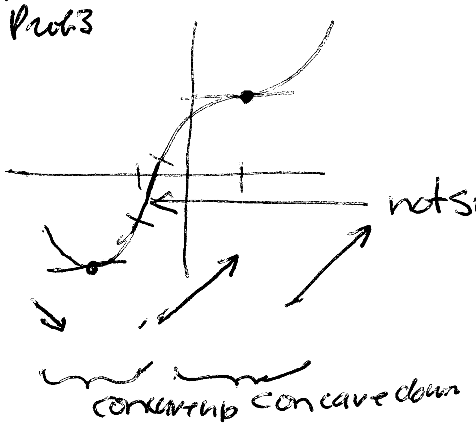


(Algebra rather complicated and lucky)

Graphs to concavity INC/DEC

3.2 Prob 3

3

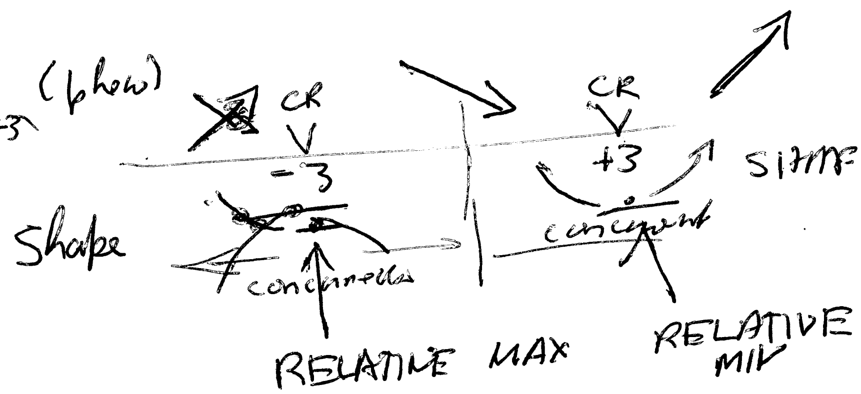


Relative Extrema @ 3.2. prob 13

$$f(x) = \frac{1}{3}x^3 - 9x + 2$$

$$\frac{dy}{dx} = x^2 - 9 = (x-3)(x+3) \quad (\text{plow})$$

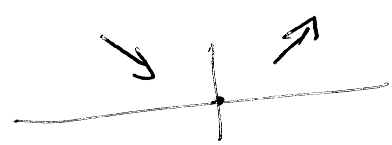
$$\frac{d^2y}{dx^2} = 2x$$



prob. 23

$$g(x) = \sqrt{x^2+1}$$

Nothing special

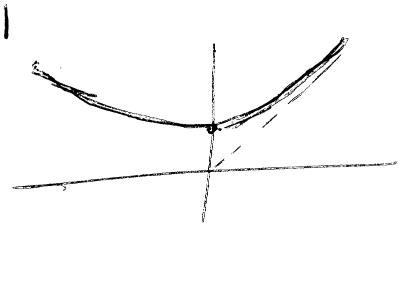


$$g'(x) = \frac{1}{2\sqrt{x^2+1}} \cdot 2x = \frac{x}{\sqrt{x^2+1}}$$

$$g''(x) = \frac{\sqrt{x^2+1} \cdot 1 - x \cdot \frac{2x}{2\sqrt{x^2+1}}}{\left(\sqrt{x^2+1}\right)^2}$$

$$= \frac{x^2 + 1 - x^2}{(x^2+1)\sqrt{x^2+1}} > 0$$

(Hyperbola!)

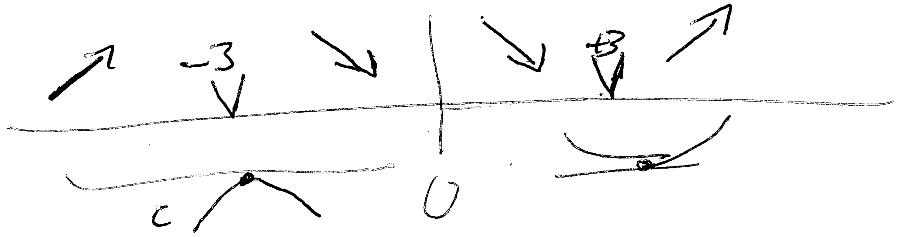


Prob 31

31. $f(x) = 2x + 1 + \frac{18}{x}$ SPECIAL $\underline{x=0}$

$\frac{dy}{dx} = 2 - \frac{18}{x^2}$; ... $x = \pm 3$

$\frac{d^2y}{dx^2} = +\frac{36}{x^3}$ changes sign at $x=0$



Best Example!

Curve Divide Regions

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REMEMBER

At CR. NO. $\frac{dy}{dx} = 0$

REL MAX if $\begin{cases} f' \text{ change } + \text{ to } - \\ \text{OR} \\ f'' < 0 \text{ (concave down} \end{cases}$

REL MIN if $\begin{cases} f' \text{ changes } - \text{ to } + \\ f' > 0 \text{ (concave up} \end{cases}$

DON'T KNOW $\{ f'' = 0$