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$\boxed{5.5}$ FTC $F' = f$ $\quad F(b)-F(a) = \int_a^b f'(x)\,dx$

Total Change $\quad F(x)-F(a) = \int_a^x f'(t)\,dt$

Find function $\begin{cases} \text{whose derivative is } 3x^2+1 \\ \text{when } x=2,\ y=6. \end{cases}$

Method $\quad F(x) = \int 3x^2+1\,dx = x^3+x+C$

solve for C $\quad F(2) = 2^3+2+C = 6 \qquad C = -2^3$

$F(x) = x^3+x-2^3 = x^3+x-8$

$F(x)-F(2) = \int_2^x (3t^2+1)\,dt$

$= t^3+t \Big|_{t=2}^{t=x}$

$\cancel{F(x) = 6+(x^3+x)-(2^3+2}$

When $x=2,\ y=6$

$F(x) = x^3+x+C$

solve for C $\quad F(2) = 6 = 2^3+2+C, \quad C=-4$

$F(x) = x^3+x-4.$

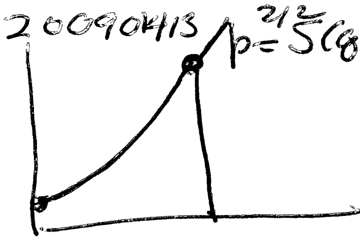
$F(x) - \underbrace{F(2)}_{6} = \int_2^x 3t^2+1\,dt$

$F(x) = 6 + (t^3+t)\Big|_{t=2}^{t=x}$

$= 6 + (x^3+x) - (2^3+2)$

$= x^3+x-4.$

Producer's Surplus



: ~~price requires to produce next~~ ($q+1^{st}$ ~~q^{th}~~ unit) price required to produce q units

= marginal Willingness to Supply

$S(q_0)$ "price at which q_0 units will be supplied"

Total amount producers receive to supply q_0 unit

$$= \int_0^{q_0} S(q)\,dq$$

$$PS = \text{Producer's Surplus} = p_0 q_0 - \int_0^{q_0} S(q)\,dq$$

5.5#12: $S(q) = 10 + 15e^{0.03q}$, $q_0 = 4$

- price at which q_0 will be supplied

$1\cancel{0+0.05}$ $\quad 10 + 15e^{.12}$

$= \dots$

- Willingness to supply

$$\int_0^{q_0} 10 + 15e^{0.03q}\,dq$$

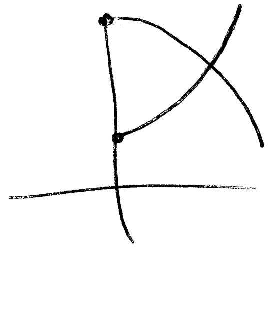
$$= 10q + \frac{15}{.03}e^{0.03q}\Big|_{q=0}^{q=q_0=4}$$

$$= \left(10\cdot 4 + \frac{15}{.03}e^{.12}\right) - \underbrace{\left(0 + \frac{15}{.03}e^{0.03q}\right)}_{\neq 0}$$

5.5 $\boxed{21.}$ Revenue at rate $R'(t) = 6{,}025 - 8t^2$

(cost) $C'(t) = 4681 + 13t^2$

$$P'(t) = R'(t) - C'(t) = R'(t) - C'(t)$$
$$= 1311 - 21t^2$$



"Begins to decline when $R' - C' < 0$"

Profit begins to decline when 20090413 3/

$\underline{P'(t) = R'(t) - C'(t) \le 0}$, immediately

$R'(t) = 7250 - 18t^2$

$C'(t) = 3620 + 12t^2$

$P'(t) = 72\cancel{50}$

$= 3630 - 30t^2$

$\cancel{t} = 0$ when $\boxed{t = 11}$

__Useful liftime__ $P(t) > 0$

$$P(t) = \int_0^t R'(s) - C'(s)\,ds$$

$$\cancel{P(t) = \sum \Delta t}$$

$$P(t) - \underbrace{P(0)}_{0} = \sum P'(\cancel{t})\Delta t$$

$$= \int_0^t 3630 - 30s^2\,ds$$

$$P(t) = 3630t - 30\frac{t^3}{3} - (\quad 0 \quad)$$

$P(t) = 0$ when $t(3630 - 10t^2) = 0$

$t = 0$ and $t^2 = \frac{3630}{10}$

$t \approx 19$ years

__Concept__ Total Profit $= \int_0^t$ Profit'(s) ds

(= integral of time rate of change

wrt.

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5.5-$\overline{28}$ Constant income 1200/yr for 5yrs 5% CC

$$PV = \int_0^5 1200\, e^{.05(5-t)}\, dt$$

5.5-30 Profit at rate of 10000 \$/yr 10yrs

4% CC P.V.

$$P.V. = \int_0^{10} 10000\, e^{-.05(10-t)}\, dt$$

Priced at

$\boxed{33}$ CS $p = \cancel{10}\ 110 - q$

$$C(q) = q^3 - 25q^2 + 2q + 3000$$

(a) Profit from first q units at p

$$P = pq - C(q)$$

Maximize profit $\cancel{P'(q) = 0}$ when

$$P(q) = (110-q)q - [q^3 - 25q^2 + 2q + 3000]$$

$$P'(q) = 110 - 2q - 3q^2 + 50q - 2$$

$$\cancel{= 100}\ 108 + 48q - 3q^2$$

$$= 3(36 + 12q - q^2)$$

$$q^2 - 12q - 36 = 0$$

$$(q-6)^2 \ldots \quad \boxed{q = 6}$$

(not very interesting

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Checkup 3. Area between $y = x + \sqrt{x}$, x axis $x=1$, $x=4$

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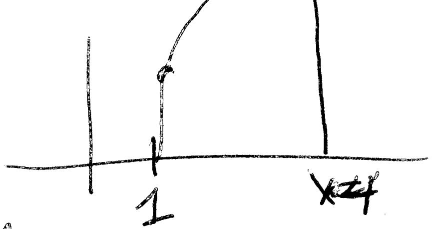

$$\int_1^4 x + \sqrt{x}\, dx$$

Check

7 Consumer's Su $p = 25 - q^2$

q (hundred items)

$p = 25 - q^2$ $(0 \le q \le 5)$

Production $q = 4$

WS. and $\int_0^4 25 - q^2\, dq = 25q - \frac{q^3}{3}\Big|_{q=0}^{4}$

$= 100 - \frac{64}{3} = 100 - 21\frac{1}{3}$

$R = pq = 9 \cdot 4 = 36$

$CS = 100 - 21\frac{1}{3} - 36$

Income Stream #7

$\boxed{72}$ $R = 5000$ \$/yr 5% CC FV at $t=3$

$$\int_0^3 5000\, e^{.05(5-t)}\, dt$$

$\boxed{73}$ $R = 1200$ 8% CC FV $t=5$

PV

$\boxed{74}$ 1000\$/y 10 yrs 7% CC