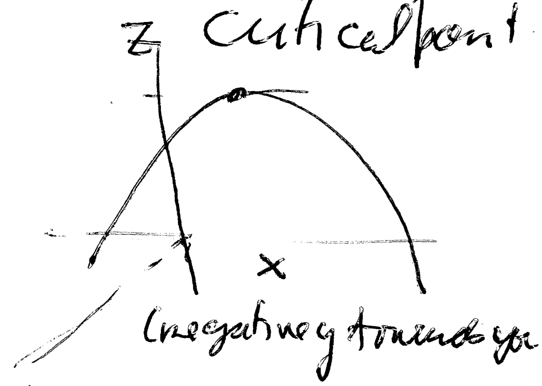
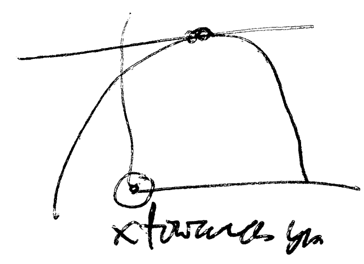
Maximum of $F(x,y)$ (two [or more]) variables

Must have



$$\frac{\partial f}{\partial x}=0$$

$$\frac{\partial f}{\partial y}=0$$



~~Has~~ two simultaneous eqns – "hard" if not "obvious"

Second Derivatives

Looks like $f_{xx}<0$ $\quad f_{yy}<0$

Not enough: OR SPECIAL: edge, vertex

Combination of 2nd derivatives discriminant

$$D \equiv f_{xx}f_{yy}-(f_{xy})^2$$

$$D \begin{cases} >0 & \text{local MAX/MIN according to } f_{xx} \text{ and } f_{yy} \text{ both} \neq 0 \\ <0 & \text{saddle – max in some direction, min in others} \\ =0 & \text{test fails – anything can happen} \end{cases}$$

Special Case: Quadratic

$$z = Ax^2 + Bxy + Cy^2 + Dx + Ey + F$$

Critical ~~Number~~ Point

$$\begin{cases} z_x = 0 = 2Ax + By + D \\ z_y = 0 = Bx + 2Cy + E \end{cases}$$

$z_{xx} = 2A$, $z_{yy} = 2C$, $z_{xy} = z_{yx}$ B

$\text{Disc} = 4AC - B^2$ (familiar)

$4AC - B^2 > 0$; $B^2 - 4AC < 0$
some quadratic has no real roots and does not change sign

$4AC - B^2 < 0$ quadratic has sign change

$4AC - B^2 = 0$ "touch" about anything

7.3: 1, 2, 3, 4, 6, 11, 12

$\boxed{4}$ $z = x^2 + 2y^2 - xy + 14y$

$z_x = 2x - y$

$z_y = \cancel{4y - x} + 14$

$4y - x + 14$

CRIT NO

$2x - y = 0$

$\cancel{4y}$ $-x + 4y = -14$

$\cancel{y = 2x}$; $2x = y$

$-x + 4 \cdot 2x = -14$

$\cancel{x = -\frac{14}{7}}$ $\quad x = -2$

$\cancel{y = -\frac{28}{9}}$ $\quad y = -4$

Elim $2x - y = 0$

$x + 4y = -14$

$2x - y = 0$

②-② $0x - \frac{7}{2}y = -14$

$\boxed{y = -4}$, $\boxed{x = -2}$

Disc $\begin{cases} z_{xx}=2 \\ z_{yy}=4 \\ z_{xy}=-1 \end{cases}$ DISC $(2)(4)-(-1)^2=7>0$

z_{xx}, z_{yy} both pos. relative MAX

$\boxed{\#12}$ $z=(x-4)\ln(xy)$ NB $xy>0$

$z_x = 1\cdot\ln(xy) + (x-4)\cdot\frac{1}{xy}\,y$

$z_y = (x-4)\cdot\frac{1}{xy}x$

Any hope? $\ln(xy) + \frac{x-4}{x} = 0$

$\frac{x-4}{y} = 0$

Solution $\boxed{x=4}$ $\ln(4y)+0=0$

(quite artificial) $4y=1,\ y=\frac{1}{4}$

~~z_{xx}~~ $z_x = \ln(x)+\ln(y)+1-\frac{4}{x}$

$z_y = \frac{x}{y} - \frac{4}{y}$

$z_{xx} = \frac{1}{x} + \frac{8}{x^2}$

$z_{yy} = (x-4)\left(-\frac{1}{y^2}\right) = 0!$

~~z_{xx}~~ $z_{xy} = \frac{1}{y}$

Disc $= 0 - \left(\frac{1}{y}\right)^2 < 0$

SADDLE

~~Add 7.2.21~~

Example 7.3.3 $f(x,y) = x^3 - y^3 + 6xy$

CRITICAL POINTS

$Z_x = 3x^2 + 6y$

$Z_y = 6x - 3y^2$

RATHER SPECIAL

$3x^2 = -6y$ $(y \le 0)$

$6x = 3y^2$ $(x \ge 0)$

$2x = y^2$

Special Algebra

7.3.4*

Add ~~7.3.24~~ 7.3.21

Frozen apple juice:

local cost 1 30 at price x: $q_1 = 70 - 5x + 4y$

natl cost 2 40 at price y: $q_2 = 80 + 6x - 7y$

Maximize Profit

$$P(x,y) \equiv (x-30)(70-5x+4y) + (y-40)(80+6x-7y)$$

Principal Part (quadratic) $-5x^2 + 4xy - 7y^2 + 6xy$

$$P_x = (70-5x+4y) + (x-30)(-5)$$

Product rule

$$+ (80+6x-7y) + (y-40)(-7)$$

$$P_y = (x-30)\cdot 4 + (80+6x-7y) + (y-40)(-7)$$

$= \ldots P_x = \ldots -10x + 10y - 20$

$P_y = 10x - 14y + 240$

$-10x + 10y = 20$

$10x - 14y = -240$

$-4y = -220$

(black box)

$\boxed{\begin{array}{l} y = 55 \\ x = 53 \end{array}}$

Note that $P_{xx} = -10$ $P_{xy} = 6$ 20090427 31

$P_{yy} = -14$

$\boxed{Disc} = \quad < 0$. Local and global max

N.B $q_1 = 70 - 5 \cdot 53 + 4 \cdot 55 \approx$

$q_2 = 80 + 6 \cdot 53 - 7 \cdot 55 \approx$

Work Problem 21. $\square$ Each cost 2

~~Profit Not q_1 =~~

(x) Duncan q_1 $(x-2)\,[40 - 50x + 40y]$

q_2 $(y-2)\,[20 + 60x - 70y]$

$P_1 + P_2 = x^2\{-50\}$

$xy\{40\} + 60\}$

$y^2\{-70\}$

$x \cdot (\cancel{40 - 120})\ (40 + 100 - 120)$

$y \cdot (\cancel{-80 + 20})\ (-80 + 20 + 140)$

constant $(-2 \cdot 40) + (-2) \cdot 120$

$P_x = \cancel{-50x} - 100x + 40y + 20$

$P_y = \quad 100x - 140y + 80$

$P_{xx} = (-100)$ $P_{xy} = 100$ $\boxed{Disc > 0}$

$P_{yy} = (-140)$

20090427 6/

$$-100x + 40y = -20$$
$$100x - 140y = -80$$

$$-100y = -100$$
$$\boxed{y = 1} \ ?$$
$$100x = 60$$
$$x = .60$$

Profit doubtful