Math 165: Optimizing Average Cost

To view animations:
`http://www2.math.uic.edu/~lewis/math165/math165avgcost.htm`.

Marginal Analysis Criterion for Minimal Average Cost

Hoffmann/Bradley, p. 243

$C(q)$ is the total cost of producing the first q units.

The average cost, $A(q)$, of producing the first q units, is

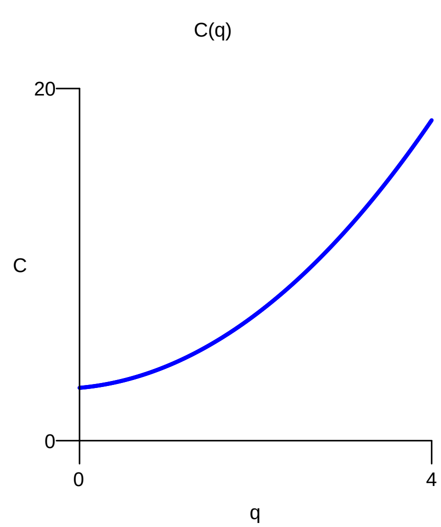
$$A(q) = C(q)/q.$$

Marginal Analysis Criterion for Minimal Average Cost. Average cost is minimized at the level of production where average cost equals marginal cost; that is $A(q) = C'(q)$.

Here is a graphical explanation of this criterion:

For a typical C(q) [Frank/Bernanke, pp. 12 ff], the Marginal Cost, $\frac{dC}{dq}$, is increasing, or the graph of $C(q)$ is concave up!

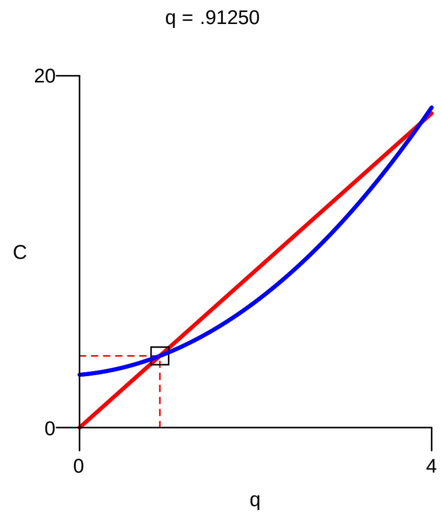
Here is the graph of a typical $C(q)$:



The average cost, $A(q)$, of producing the first q units, is

$$A(q) = \frac{C(q)}{q}.$$

The average cost, A(q), may be interpreted as the slope of the line through $(0,0)$, and $(q, C(q))$.



The (moving) box represents the point on the cost curve, the slope of the (moving) red line represents the average cost.

Notice that the slope of the red line is minimized when the line is tangent to the graph of C(q); i.e. A(q) = dC/dq.

Note that the condition

$$\frac{C(q)}{q} = \frac{dC}{dq}$$

is the same as

$$1 = \frac{q}{C}\frac{dC}{dq}.$$

The quantity $C_E = \dfrac{q}{C}\dfrac{dC}{dq}$ is the *elasticity of cost with respect to output* or *output–elasticity of cost*.

Then

$$\text{percentage change in Total Cost } C \approx C_E \cdot \text{percentage change in output } q.$$

When $0 < C_E < 1$ (relatively inelastic), $A(q)$ is decreasing; when $1 < C_E$ (relatively elastic), $A(q)$ is increasing.