<u>MTHT 430, 8/29/07</u> William Howard substituting for Jeff Lewis.

P1 - P12: axioms for an ordered field.

Your main goal for Chap. 1 is to develop the ability to prove various familiar properties of real numbers from those axioms. In other words, show these properties are theorems ~~of [illegible]~~ for an ordered field. Engineers, statisticians, etc., do not need to know how to do this; but teachers do. It is about the organization of knowledge.

In your proofs, always treat addition and multiplication as binary operations. Hence don't write $a+b+c$; see Spivak p.3. It is OK to write $r+st$ because, by convention, the multiplication is done first (so $r+st$ means $r+(st)$). This applies to Spivak's problems 1(i)-(iii), 2, 3, 5-8, 12. In other problems, he is ~~g~~ already going beyond what I called "your main goal".

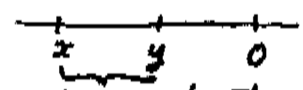
<u>Definitions</u> $a-b$ denotes $a+(-b)$

$\frac{a}{b}$ " ab^{-1}. Thus the theorem

$\frac{a}{b} = \frac{ac}{bc}$ $(b, c \neq 0)$ says $ab^{-1} = (ac)(bc)^{-1}$ (Spivak, problem 3(i))

Absolute value: definition: $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0. \end{cases}$

$x \leq y$ means: $x < y$ or $x = y$ (in mathematics, "or" is always taken in the inclusive sense).

Geometric meaning of $|x-y|$: 

$|x-y|$ is this <u>distance</u>

"Triangle inequality": $|a+b| \leq |a| + |b|$ (Spivak p.11).

Often used in the form $|x-z| \leq |x-y| + |y-z|$.