MthT 430 Chapter 10a Projects – Derivatives
In Class November 28, 2007

1. Let $F(x)$ be a function such that

   - domain$(F) = \mathbf{R}$.
   - For all x, y, $F(x+y) = F(x) \cdot F(y)$.
   - $F(0) \neq 0$.
   - F is differentiable at 0 and $F'(0) = \pi$.

   Show that, for every a, F is differentiable at a and find a formula for $F'(x)$. Here *formula* is an expression in terms of F or a familiar function.

2. Let $G(x)$ be a function such that

   - domain$(G) = \mathbf{R}^+ \equiv \{x \mid x > 0\}$.
   - For all $x, y > 0$, $G(x \cdot y) = G(x) + G(y)$.
   - $G(1) = 0$.
   - G is differentiable at 1 and $G'(1) = 1$.

   Show that, for every $a > 0$, G is differentiable at a, and find a formula for $G'(x)$, $x > 0$. Here *formula* is an expression in terms of G or a familiar function.

3. Let E be a function such that

   - E is differentiable for all x,
   - E is an even function.

   Show that

   - E' is an odd function,
   - $E'(0) = 0$.

4. S and C are functions such that

   - For all x, S and C are differentiable,
   - $S' = \pi C$ (for all x, $S'(x) = \pi C(x)$), $C' = -\pi S$.
   - $S(0) = 0$, $C(0) = \pi$.

   Find a formula for $S^{(n)}(0)$, $n = 0, 1, 2, \ldots$