

## MthT 430 Chapter 5a Projects – Limits

### Limits

1. Let  $F(x)$  be a function such that

- $\text{domain}(F) = \mathbf{R}$ .
- For all  $x, y$ ,  $F(x + y) = F(x) \cdot F(y)$ .
- $F(0) \neq 0$ .
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$$\lim_{x \rightarrow 0} \frac{F(x) - F(0)}{x} = \pi.$$

Find

$$\lim_{x \rightarrow 0} \frac{F(a + x) - F(a)}{x}.$$

2. Let  $G(x)$  be a function such that

- $\text{domain}(G) = \mathbf{R}^+ \equiv \{x \mid x > 0\}$ .
- For all  $x, y > 0$ ,  $G(x \cdot y) = G(x) + G(y)$ .
- $G(1) = 0$ .
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$$\lim_{x \rightarrow 0} \frac{G(1 + x)}{x} = \pi^2.$$

For  $a > 0$ , find

$$\lim_{x \rightarrow 0} \frac{G(a + x) - G(a)}{x}.$$

3. Let  $f(x)$  be a function such that

- $\text{domain}(f) = [0, 1)$ .
- For all  $x$  (in  $[0, 1)$ ),  $0 \leq f(x) < 1$ .
- The function  $f$  is increasing on  $[0, 1)$ .

Show that there is a number  $L$ ,  $0 \leq L \leq 1$ , such that

$$\lim_{x \rightarrow 1^-} f(x) = L.$$

**Hint:** Construct a binary expansion for  $L$ .