MthT 430 Projects Chapter 6a Limits and Continuity

Limits

1. (Omit for now – will discuss after MidTerm Assessment) Let $f(x)$ be a function such that

   - domain $(f) = [0, 1)$.

   - For all x (in $[0, 1)$), $0 \leq f(x) < 1$.

   - The function f is increasing on $[0, 1)$.

   Show that there is a number L, $0 \leq L \leq 1$, such that

   $$\lim_{x \to 1^-} f(x) = L.$$

   Hint: Construct a binary expansion for L.

2. Discuss the continuity of the function described on p. 97 and whose graph is sketched in FIGURE 14.

3. Prove: If g is continuous at a, $g(a) \neq 0$, then there is a $\delta > 0$ for which $(a - \delta, a + \delta)$ is contained in the domain of $\dfrac{1}{g}$.

4. Spivak, Chapter 6, Problem 3.

5. Spivak, Chapter 6, Problem 13.

6. If f is continuous at 0, $f(0) = 0$, $g(x)$ is defined for all x near 0, and $|g|$ is a bounded function, say all you can about
   $$\lim_{x \to 0} f(x) \cdot g(x).$$