MthT 430 Notes Chap7a Three Hard Theorems

(CFIVP) Continuous Functions on Intervals Have the Intermediate Value Property

Theorem 1. *If f is continuous on $[a,b]$ and $f(a) < 0 < f(b)$, then there is some x in $[a,b]$ such that $f(x) = 0$.*

An argument constructing the binary expansion for one such x will be given in class. See

`http://www.math.uic.edu/~lewis/mtht430/chap7b.pdf`

(CFCIB) Continuous Functions on Closed Intervals are Bounded

Theorem 2. *If f is continuous on $[a,b]$, then f is bounded above on $[a,b]$, that is, there is some number N such that $f(x) \leq N$ for all x in $[a,b]$.*

CFCIMAX) Continuous Functions on Closed Intervals assume a Maximum Value for the Interval

Theorem 3. *If f is continuous on $[a,b]$, then there is a number y in $[a,b]$ such that $f(y) \geq f(x)$ for all x in $[a,b]$*

Consequences

- If f is continuous on $[a,b]$ and changes sign, then the equation $f(x) = 0$ has a root in (a,b).
- (**Intermediate Value Property for Continuous Functions on Closed Intervals**) If f is continuous on $[a,b]$ and ξ is between $f(a)$ and $f(b)$, then the equation $f(x) = \xi$ has a root in (a,b).
- Every nonnegative number ξ has a unique nonnegative square root, denoted $\sqrt{\xi}$, which satisfies $\sqrt{\xi} \geq 0$ and $\left(\sqrt{\xi}\right)^2 = \xi$.