Ratio and Proportion

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1 Ratio and Proportion

A chestnut If a chicken and a half lays an egg and a half in a day and a half how many hens does it take to lay six eggs in six days?

Descriptions of Proportions Here are a number of ways of describing each of two possible relationships.

1. direct variation
   (a) $y_1 : x_1 :: y_2 : x_2$
   (b) $y = kx$
   (c) graph is a straight line through the origin
   (d) $f(cx) = cf(x)$
   (e) similar triangles
2. inverse variation
   (a) $y_1 : x_1 :: x_2 : y_2$
   (b) $y = k/x$
   (c) graph is a hyperbola
   (d) $f(cx) = f(x)/c$
   (e) What is a good geometric representation?

Concrete meanings for proportionality constants Consider a family of rectangles each with area 60 sq. in. Then, height varies inversely with respect to length.

> Would (should?) a student understand the following: Consider a rectangle with area 60 sq. in. The height varies inversely with the length.

Old proportion problems Here are some problems taken from an arithmetic book published in 1892 and reissued in 1895, 1911, and 1920. Which of the approaches above provides the most elegant - the most understandable- solution of each problem?

1. The first group are from a section called simple proportion.
   (a) If the interest upon a sum of money for 9 months is 318.69, what will be the interest for 11 1/2 months?
   (b) If 15 men can do a piece of work in 36 days, in how many days can they perform the same work with the assistance of 9 men more?
   (c) If a garrison of 200 men has provisions for 8 months, how many men must leave at the end of 5 months that the provisions remaining may last the rest 8 months longer?
2. More complicated problems are compound proportions.
   (a) If 11 men build 45 rods of wall in 6 days of 10 hours each, how many men will be required to build 81 rods of wall in 12 days of 11 hours each?
   (b) (oral exercise) If 7 men can dig 32 rods of ditch in 1 day, how many men will be required to dig 92 rods in 3/4 day.

The last problem is arcane at best but what does it mean? Rods of fence built is directly proportional to number of men and number of days and number of hours per day. Part b) can be expressed as the number of men is directly proportional to the length of ditch and inversely proportional to the number of days taken.

The volume of a gas varies directly with number of moles of gas present and with the temperature but inversely with the pressure. This is represented by the ideal gas law.

$$PV = nRT$$

This gives rise to problems of the following sort.

2 The gas laws

Exercises 32 gm of oxygen occupy 22,414 cc^3 at a pressure of one atmosphere and a temperature of 273 degrees Kelvin. Volume of a gas varies directly with the temperature (in degrees Kelvin) and inversely with the pressure.

1. What is the effect on the volume of a given quantity of oxygen of
   (a) doubling the pressure?
   (b) doubling the temperature? tripling the temperature?
2. What is the volume of 32 gm of oxygen at a temperature of 273 degrees Kelvin and
   (a) twice atmospheric pressure?
   (b) 1.5 times atmospheric pressure? a pressure of 2.73 atmospheres?
3. The proportionality constant:
   (a) Write a formula for volume of a gas in terms of pressure, temperature and a proportionality constant.
   (b) What is the value and what are the dimensions of the proportionality constant?
   (c) 1 atm (one atmosphere) is 76 cm of mercury. What is the proportionality constant if one of the dimensions is cm of mercury - is inches of mercury?
4. Use the memory key on your calculator and the proportionality constant calculated above to solve the following problems. What is the volume of 32 gm of oxygen at
   (a) a temperature of 273 degrees Kelvin and twice atmospheric pressure?
   (b) a temperature of 400 degrees Kelvin and 1.5 times atmospheric pressure?
   (c) a temperature of 4500 degrees Kelvin and a pressure of 2.73 atmospheres?
5. Is the volume of a gas at standard pressure directly proportional to its temperature in Fahrenheit, in Celsius?