MATH 121
Final Exam
Study Guide

This is a list of problems to help you prepare for the final exam. It is not considered to be exhaustive and you should not expect to find your actual exam problems in the list below. It is to serve as a study aid and it cannot be a substitute for in-class reviews or study of your class notes.

1. Find all solutions to the equation:

$$3x^2 - 5x - 2 = 0$$

2. Find the solution to the equation:

$$x^3 - 2x = x^2 - 1$$

that lies in the interval $[0, 1]$.

3. The area of a right triangle is 24 and the hypotenuse has length 10. What are the lengths of the remaining sides?

4. A rectangle of perimeter 12 is inscribed in a circle of radius $\sqrt{5}$. What are the dimensions of the rectangle?

5. A train going from Chicago to Cleveland leaves at 10AM and travels at a speed of 60 miles per hour. A second train going from Cleveland to Chicago leaves at 12PM and travels at a speed of 55 miles per hour. If the cities are 500 miles apart, at what time do the trains meet and how far from Chicago do they meet?

6. Two cars leave from the same point. One car heads north at a speed of 30 miles per hour. The other car heads east at a speed of 35 mph. How far apart are the two cars after 30 minutes?

7. Compute the difference quotient, $\dfrac{f(x+h) - f(x)}{h}$, for the function $f(x) = x^2 + x$.

8. Find the domain of the function $f(x) = \dfrac{x-1}{\sqrt{4-x^2}}$.

9. Let $f(x) = x^2$ and $g(x) = x + \sqrt{x+2}$.

   (a) What is the domain of $g(x)$?

(b) Write the rule for the composite function $g \circ f$.

10. Find the inverse of the function $f(x) = \dfrac{2x-5}{7x+4}$.

11. Find the inverse of the function $f(x) = 3^{2x-1}$.

12. Find the remainder when the function $f(x) = x^{20} - 3x^{13} + 7x + 1$ is divided by the function $g(x) = x + 1$.

13. Show that the function $g(x) = x^2 + 1$ is a factor of the function $f(x) = x^5 + x^2 - x + 1$.

14. Let $f(x) = x + \sqrt{x-1}$. Write the rule of the function $g(x)$ obtained by performing the following transformations on $f(x)$:

    I. shift 1 unit upward

    II. expand vertically by a factor of 3

    III. shift 2 units to the left

    IV. reflect across the x-axis

15. Let $f(x) = x-1$ and $g(x) = -3(x+2)+5$. Describe a series of transformations that transforms $f(x)$ into $g(x)$.

16. Let $f(x) = x^2 + 2^x$ and $g(x) = 2(x-1)^2 + 2^x$. Describe a series of transformations that transforms $f(x)$ into $g(x)$.

17. Find all roots (real and complex) of the function $f(x) = 6x^4 + 7x^3 - x^2 - 2x$.

18. Find all roots (real and complex) of the function $f(x) = 2x^3 - 9x^2 + 14x - 5$.

19. For the function $f(x) = \dfrac{x}{x+2}$, find:

    (a) the x-intercept

    (b) the y-intercept

    (c) the vertical asymptote(s)

    (d) the horizontal asymptote

20. For the function $f(x) = \dfrac{(x-1)^2(x+2)}{(x-1)(x+2)^2}$, find:

    (a) the roots of $f(x)$

    (b) the vertical asymptote(s)

    (c) the hole(s)

    (d) the horizontal asymptote

21. Solve the inequality:
$$x + 1 \leq 3x - 2 < 4x + 2$$

22. Solve the inequality:
$$\frac{x-1}{2x+3} \leq 1$$

23. Simplify the following expressions:
    (a) $\dfrac{1-i}{1+i}$ (write in the form $a + bi$)
    (b) i^{87}
    (c) $\left(\dfrac{x^2}{y^5}\right)^{2/3} \sqrt[3]{x^3 y^{-7}}$
    (d) $\ln(x+1) - \ln(2x) + 1$
    (e) $\log 10^{x^2+1} - 2\log 10 - x^2$
    (f) $e^{\ln(x+1)} - e^{2\ln x}$

24. Suppose you invest \$1000 in a savings account with an interest rate of 4%. How much money is in the account after 2 years if the interest is compounded
    (a) quarterly?
    (b) monthly?
    (c) daily?
    (d) continuously?

    Write your answers to two decimal places.

25. At what interest rate should \$5000 be invested so that there is \$5250 in the account after 1 year if the interest is compounded
    (a) monthly?
    (b) continuously?

    Write your answers as a percentage and to two decimal places.

26. The growth in height of trees is frequently described by a logistic equation. Suppose the height h (in feet) of a tree at age t (in years) is
$$h = \frac{120}{1 + 200e^{-0.2t}}$$
    (a) What is the height of the tree at age 10?
    (b) At what age is the height 50 feet?

27. If a language originally had N_0 basic words of which $N(t)$ are still in use, then

$$N(t) = N_0(0.805)^t$$

where time t is measured in millennia (1 millenium = 1000 years). After how many years are one-half the basic words still in use?

28. Solve the following equation:

$$2^{x-1} = 4^{x-2}$$

Your answer should be an integer.

29. Find the exact solution to the following equation:

$$3^x = 5^{x-3}$$

30. Find all possible solutions to the following equation:

$$\ln x - \ln(2x-1) = \ln 3$$

31. Find all possible solutions to the following equation:

$$2\log x - \log(4x-5) = 0$$

32. Find the angle t that is coterminal with $-\dfrac{18\pi}{5}$ and lies in the interval $[0, 2\pi]$.

33. Find three angles that are coterminal with $\dfrac{\pi}{6}$.

34. Evaluate the following expressions:

(a) $\cos\dfrac{\pi}{4}$ (b) $\sin\dfrac{5\pi}{6}$ (c) $\tan\pi$ (d) $\sec\dfrac{5\pi}{3}$ (e) $\cos\left(-\dfrac{\pi}{2}\right)$

(f) $\cot\dfrac{\pi}{6}$ (g) $\csc\dfrac{\pi}{2}$ (h) $\sin\left(-\dfrac{\pi}{4}\right)$ (i) $\tan\dfrac{2\pi}{3}$ (j) $\sec\left(-\dfrac{3\pi}{4}\right)$

35. If $\sin x = -\frac{3}{4}$ and $\cos x < 0$ then compute:

(a) $\cos x$ (b) $\cot x$ (c) $\csc x$ (d) $\sin(-x)$

36. If $\cos x = \frac{1}{3}$ and $\frac{3\pi}{2} \le x \le 2\pi$ then compute:

(a) $\sin x$ (b) $\tan x$ (c) $\sec x$ (d) $\cos(-x)$

37. For the function:

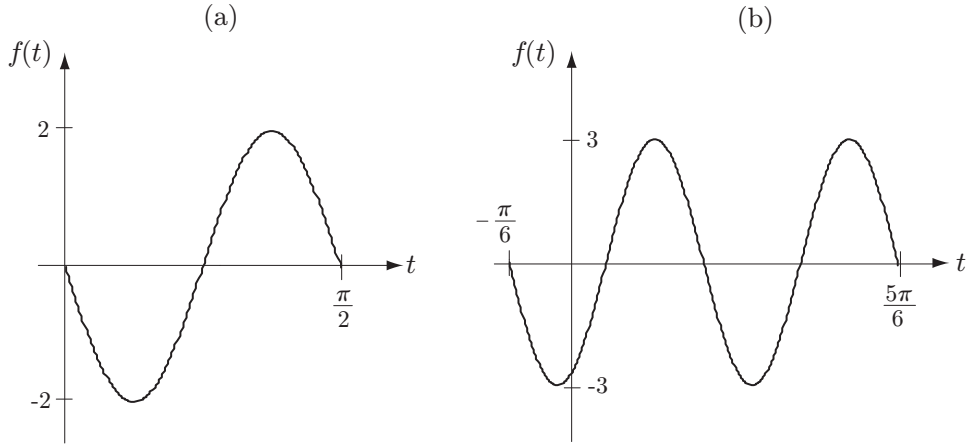
$$f(t) = -4\sin\left(5t - \frac{\pi}{2}\right)$$

(a) find the amplitude, period, and phase shift of $f(t)$

(b) sketch one period of the graph of $f(t)$

38. Suppose that the amplitude of $f(t)$ is 2, the period of $f(t)$ is 4, and the phase shift of $f(t)$ is $-\frac{\pi}{3}$. Find values of A, b, and c such that $f(t) = A\cos(bt + c)$.

39. For each graph below, determine the values of A, b, and c such that $f(t) = A\sin(bt + c)$.



40. State whether each of the following statements is true or false. Provide a short explanation of your answer.

(a) $\sin 2x = 2\sin x$

(b) $\cos x \tan x = \sin x$

(c) $\sin(x + \pi) = \sin x$

(d) $\tan x + \cot x = 1$

(e) $\cos^2 x = \cos^2 \dfrac{x}{2} - \sin^2 \dfrac{x}{2}$

(f) $\tan x = \dfrac{\cos x}{\sin x}$

41. Prove the following identity:

$$\frac{\cot\theta - \tan\theta}{\sin\theta + \cos\theta} = \csc\theta - \sec\theta$$

42. Prove the following identity:

$$\frac{1 + \csc\beta}{\cot\beta + \cos\beta} = \sec\beta$$

43. Prove the following identity:

$$\frac{\tan^2 x}{\sec x + 1} = \frac{1 - \cos x}{\cos x}$$

44. Find the exact values of:

(a) $\sin \dfrac{5\pi}{12}$ (b) $\cos \dfrac{13\pi}{12}$ (c) $\cos 15^\circ$ (d) $\sin 22.5^\circ$

45. If $\cos x = -\frac{3}{7}$ and $\frac{\pi}{2} \leq x \leq \pi$ then find

(a) $\sin x$ (b) $\cos \dfrac{x}{2}$ (c) $\sin\left(2x + \dfrac{\pi}{2}\right)$

46. Find the exact values of the following expressions:

(a) $\cos\left(\sin^{-1} \dfrac{1}{4}\right)$ (b) $\sin^{-1}\left(\tan \dfrac{2\pi}{3}\right)$ (c) $\sin^{-1}\left(\sin \dfrac{5\pi}{6}\right)$

(d) $\tan\left(\cos^{-1}\left(-\dfrac{2}{9}\right)\right)$ (e) $\csc\left(\cos^{-1} \dfrac{1}{5}\right)$

47. Find all solutions to the equation

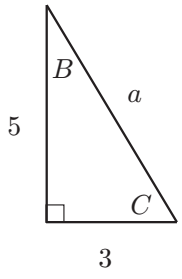
$$\cos 8x = \frac{\sqrt{2}}{2}$$

48. Find all solutions to the equation

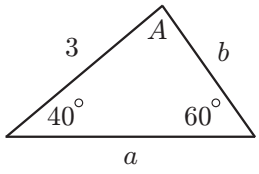
$$4\sin^2 x + 8\sin x + 3 = 0$$

49. Solve each of the triangles below

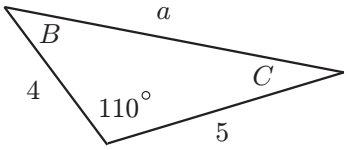
(a)



(b)



(c)



50. The angles of elevation of a hot-air balloon from two points A and B on level ground are 24° and 47°, respectively. Points A and B are 8.4 miles apart and the balloon is between the points, in the same vertical plane. Find the height of the balloon above the ground to two decimal places.

51. A baseball diamond has four bases (forming a square) that are 90 feet apart. The pitcher's mound is 60.5 feet from home plate. Find the distance from the pitcher's mound to each of the other three bases. Write your answers to two decimal places.

52. Consider the complex number $z = 1 + \sqrt{3}i$.

    (a) Compute the modulus of z.

    (b) Write z in polar form.

    (c) Compute z^{12} using DeMoivre's Theorem.

    (d) Find the fourth roots of z.

53. Find all solutions to the equation

$$x^3 = i$$

54. Let $\overrightarrow{\mathbf{u}} = \langle 2, 3 \rangle$ and $\overrightarrow{\mathbf{v}} = \langle -1, -2 \rangle$.

    (a) Compute the lengths of $\overrightarrow{\mathbf{u}}$ and $\overrightarrow{\mathbf{v}}$.

    (b) Compute $2\overrightarrow{\mathbf{u}} - 3\overrightarrow{\mathbf{v}}$.

    (c) Find the unit vector in the direction of $\overrightarrow{\mathbf{u}}$.

55. Let $P = (2, 5)$ and $Q = (-3, 1)$.

    (a) Compute $\overrightarrow{PQ}$.

    (b) Compute the length of $\overrightarrow{PQ}$.

    (c) Find the unit vector in the direction of $\overrightarrow{QP}$.